83.5/100

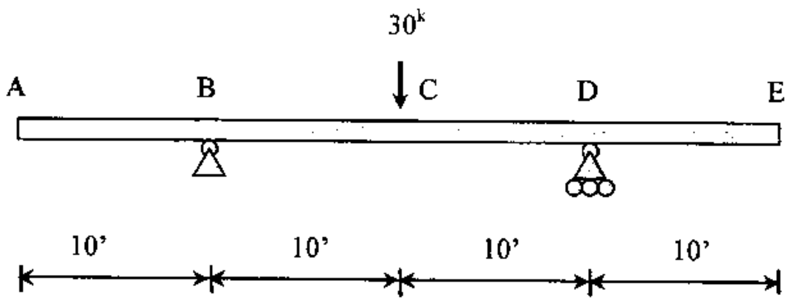
CE 461 – Structural Analysis, Fall 2006 **Name:**_
EXAM No. 2

9.5 1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

REAL BEAM		CONJUGATE BEAM	
Type of Support	**Real Slope and Deflection**	**Conjugate Shear and Bending Moment**	**Type of Support**
Simple end support	$\theta \neq 0$ $\Delta = 0$ -0.5	$V \neq 0$ ✓ $M = 0$ -0.5	Stays Same ✓
Fixed support (encastre)	$\theta \neq 0$ × $\Delta = 0$	$V \neq 0$ × $M = 0$ × -0.5	Becomes pinned × -1
Free end	$\theta \neq 0$ $\Delta \neq 0$ × -0.5	$V = 0$ $M = 0$ × -0.5	Fixed ✓
Simple interior support	$\theta = 0$ $\Delta = 0$ × -0.5	$V \neq 0$ $M = 0$	Hinge ✓
Internal hinge	$\theta = 0$ $\Delta \neq 0$	$V \neq 0$ $M = 0$ × -0.5	Hinge X -1

-5.5

2 2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?



→ 4 for B-C

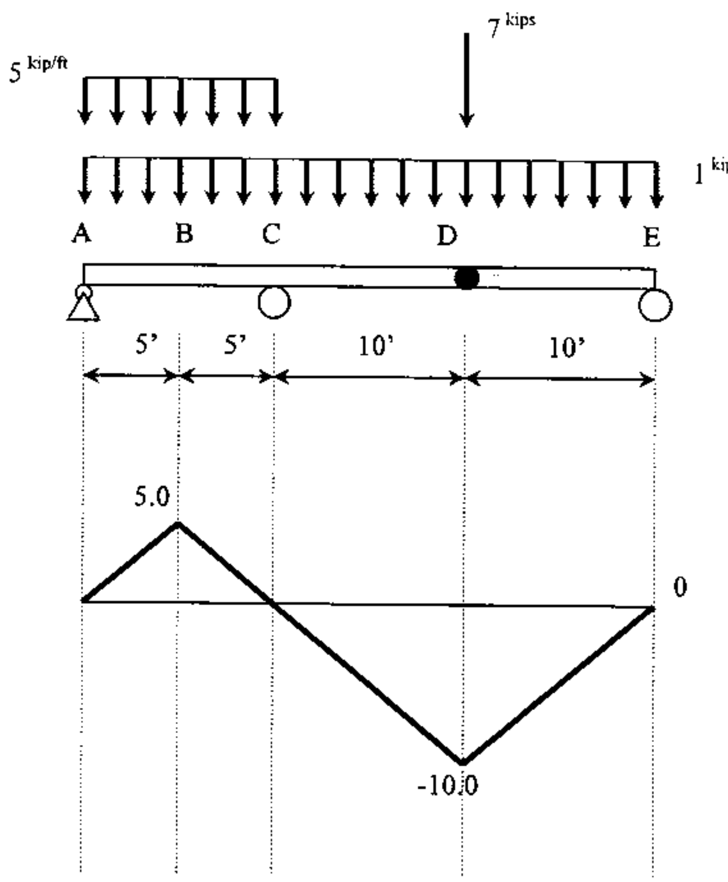
→ 4 for C-D

$\boxed{\text{8 constants of Integration}}$

what about AB & DE ?

-3

8 3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the [redacted] moment at B generated by the loads shown.



Moment @ B = $(5k/ft)(\frac{1}{2})(5)(5) + (5)(\frac{1}{2})(5)(5) + 1(\frac{1}{2})(5)(5) + 1(\frac{1}{2})(5)(5) + 1(\frac{1}{2})(10)(10) + 1(\frac{1}{2})(10)(10) + 7^k(-10^{ft})$

Live Live Dead Dead Dead

$= 62.5 + 62.5 + 12.5 + 12.5 - 50 - 50 - 70$

$\boxed{\text{Moment @ B} = +50\ ft\cdot k}$ $- 70 = 20$

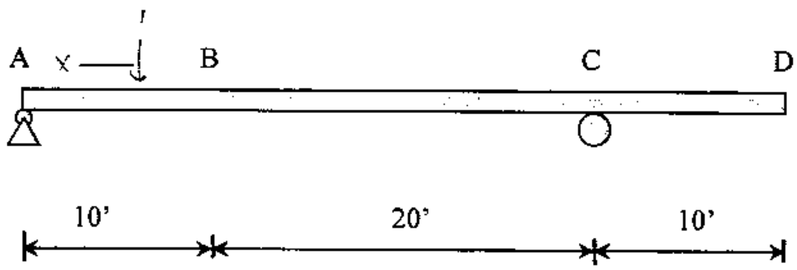
-2

30

4. [35 pts] Given the following beam, draw the influence lines for:

- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.



Equilibrium Method

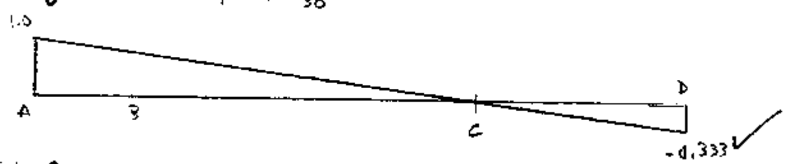
I.L A_y

$\circlearrowleft \Sigma M_A = 0 = 1(x) - C_y(30)$

$C_y = \frac{x}{30}$

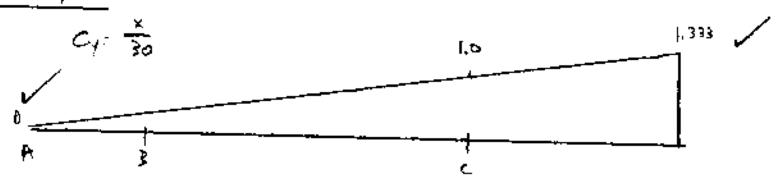
$\Sigma F_y = 0 = A_y + C_y - 1$

$A_y = 1 - \frac{x}{30}$

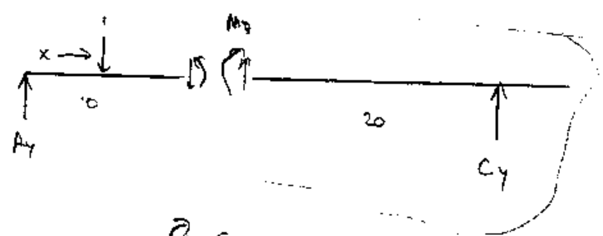


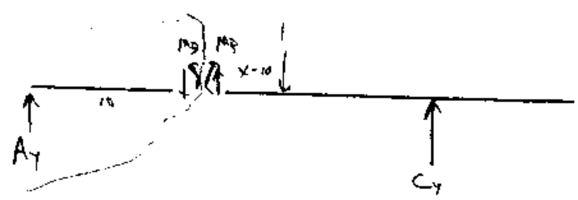
I.L. C_y

$C_y = \frac{x}{30}$



I.L. MB





$\circlearrowleft \Sigma M_B = 0 = -C_y(20) + M_B$ $\qquad$ $\circlearrowleft \Sigma M_B = 0 = A_y(10) - M_B$

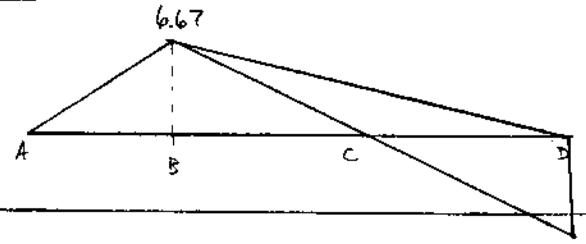
$M_B = 20C_y \quad 0 \le x \le 10$ $\qquad$ $M_B = 10A_y \quad 10 \le x \le 40$

$$M_B \begin{cases} 20C_y = \frac{20x}{30} & 0 \le x \le 10 \\ 10A_y = 10 - \frac{10x}{30} & 10 \le x \le 40 \end{cases}$$

True, but your origin for A_y started at 0, & this one starts at 10, so you must adjust. -5

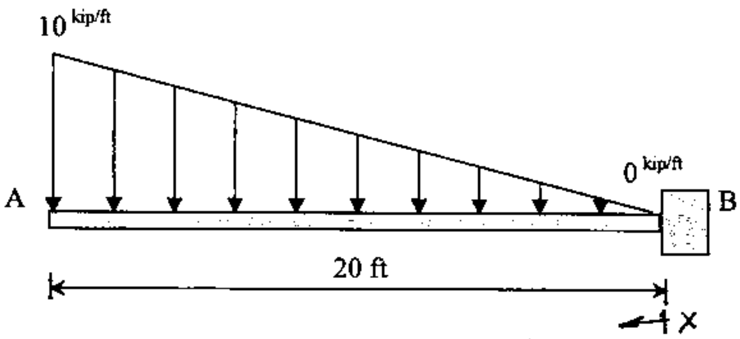
$$M_B = \begin{cases} \dfrac{20x}{30} & 0 \le x \le 10 \\ 10 - \dfrac{10x}{30} & 10 \le x \le 40 \end{cases}$$

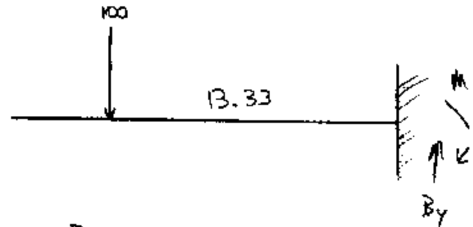
<u>I.L. For M_B</u>



29

5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let E=29,000 ksi and I=4200 in^4. Your answer should be given in inches.





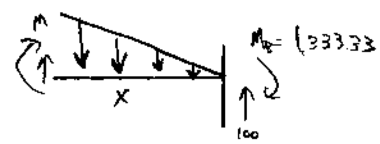
$\circlearrowleft + \sum M_B = 0 = M_B - 100(13.33)$

$M_B = 1,333.33$ ft·k

$+\uparrow \sum F_y = 0 = B_y - 100$

$B_y = 100$

Take Cut For Internal Moment



forgot to include effect of external load in moment calc

-6

$\circlearrowleft + \sum M = 0 = M + 1,333.33 - 100(x)$

$M = 100x - 1,333.33 - \frac{1}{2}(x)(\frac{1}{2}x)\underbrace{(\frac{1}{3}x)}_{\text{moment arm}}$

$EI\,\theta = \int M = \int 100x - 1,333.33 = \frac{100x^2}{2} - 1333.33x + C_1$

$\theta = (50x^2 - 1,333.33x + C_1)\frac{1}{EI}$

$y = \int \theta = \frac{1}{EI}\int 50x^2 - 1333.33x + C_1 = \frac{50x^3}{3} - \frac{1333.33x^2}{2} + C_1 x + C_2$

$y = (16.67x^3 - 666.66x^2 + C_1 x + C_2)\frac{1}{EI}$

B.C.

$\theta_{@}\ x = 0 = 0$

$C_1 = 0$

$y @ x = 0 = 0$

$C_2 = 0$

Max Def.

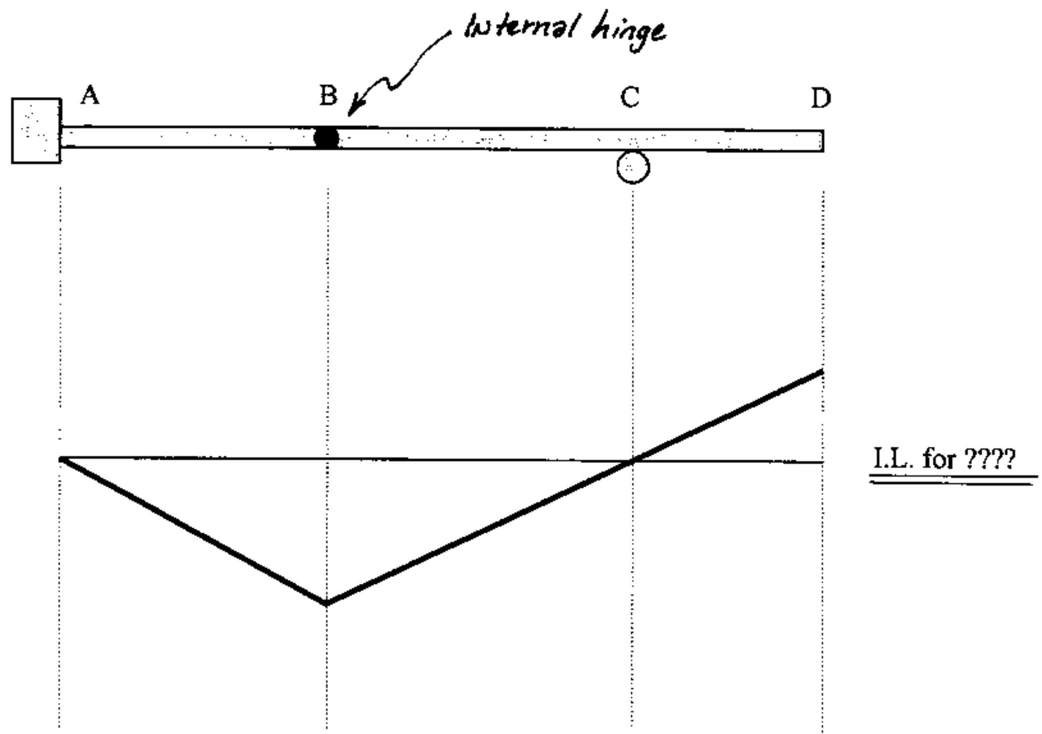
$y = (16.67(20)^3 - 666.66(20)^2)\frac{1}{EI} = \frac{-133304\ ft^3 k}{EI}$

$= -\frac{133304\ ft^3 k}{}\left|\frac{(12\ in)^3}{1\ ft^3}\right. = \frac{-230,349,312\ in^3 k}{EI}$

$= \frac{-230,349,312\ in^3 k}{(29,000\ ksi)(4200\ in^4)}$

$\boxed{y_{max} = -1.89 \text{ inches}}$

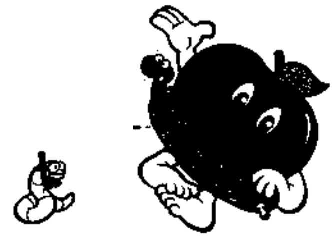
5 Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: "Influence Line for the Vertical Reaction at A".) Hint: The answer is not really "Influence Line for the Vertical Reaction at A". ☺



Influence Line For The Moment @ A

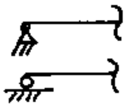
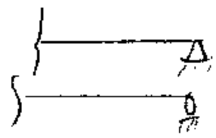
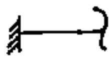
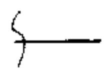
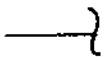
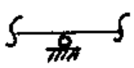
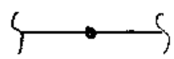
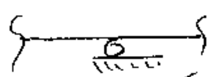
Value @ A=0

98/100 Great!



CE 461 – Structural Analysis, Fall 2006 **Name**
EXAM No. 2

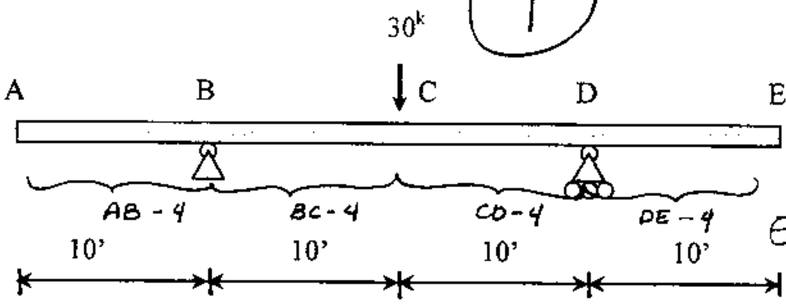
14 ~~15~~ 1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

REAL BEAM		CONJUGATE BEAM	
Type of Support	**Real Slope and Deflection**	**Conjugate Shear and Bending Moment**	**Type of Support**
Simple end support 	$\theta \neq 0$ $\Delta = 0$	✓ $V \neq 0$ ✓ $M = 0$	 Simple end support ✓
Fixed support (encastre) 	$\theta = 0$ $\Delta = 0$	✓ $V = 0$ ✓ $M = 0$	 Free end ✓
Free end 	$\theta \neq 0$ $\Delta \neq 0$	✓ $V \neq 0$ ✓ $M \neq 0$	 ✓
Simple interior support 	$\theta \neq 0$ $\Delta = 0$	✓ $V \neq 0$ ✓ $M = 0$	 Internal hinge ✓
Internal hinge 	$\theta \neq 0$ $\Delta = 0$ ✗ -0.5	✓ $V \neq 0$ $M = 0$ ✗ -0.5	 Internal Roller ✓

-1

6 2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?

4 ×



$W = 0$

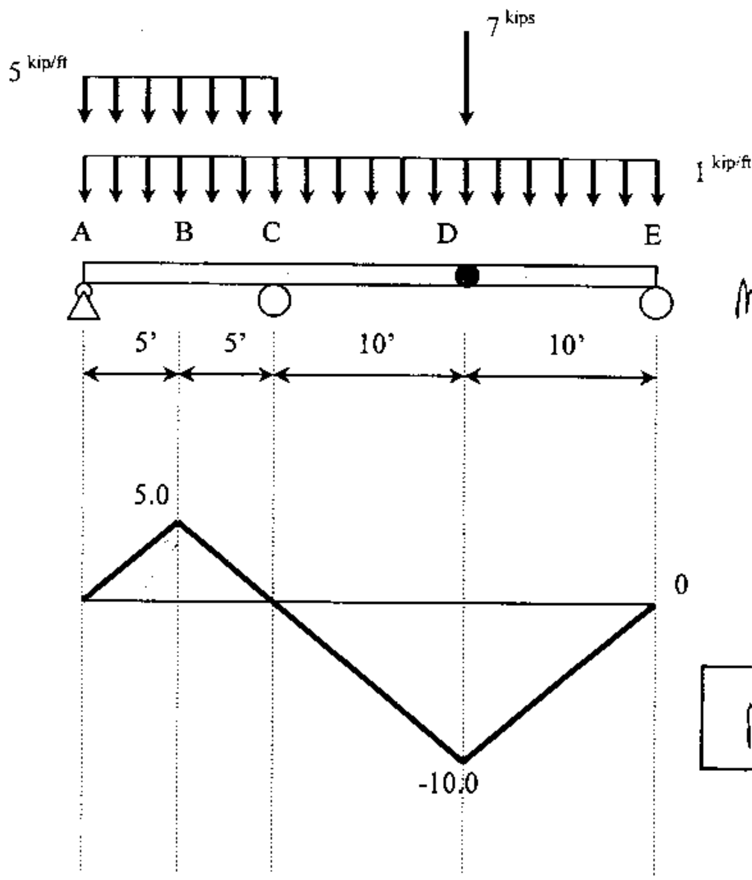
$V(x) = C_1$

$M(x) = C_1 x + C_2$

$EI\theta(x) = \frac{C_1 x^2}{2} + C_2 x + C_3$

$EIy(x) = \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$

10 3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the ~~absolute maximum (positive or negative)~~ moment at B generated by the loads shown.



$M_B = \frac{5(10)}{2}[5^{k/ft} + 1^{k/ft}]$

$- \frac{10[20]}{2}[1^{k/ft}]$

$-7[10]$

$150 - 100 - 70$

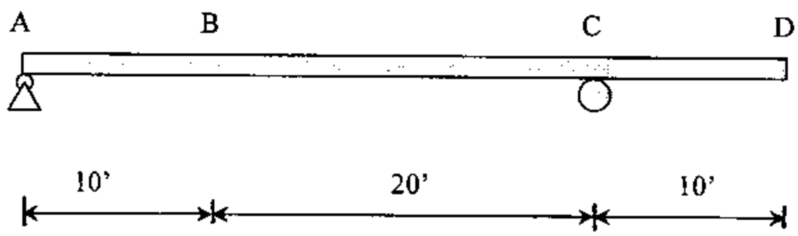
$\boxed{M_B = -20^{k \cdot ft}}$ ✓

35 - Great!

4. [35 pts] Given the following beam, draw the influence lines for:

- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

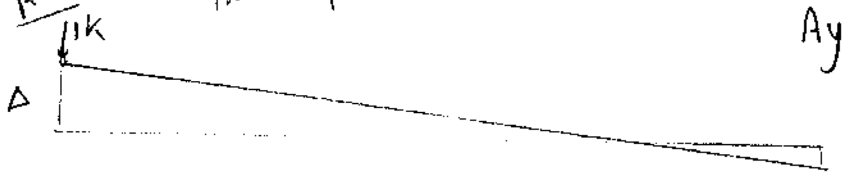


Vertical Reaction at A

Release support at A, move pos. Δ

$\Sigma F_y = 0 = A_y + C_y - 1$

$A_y + C_y = 1$



If unit load at A

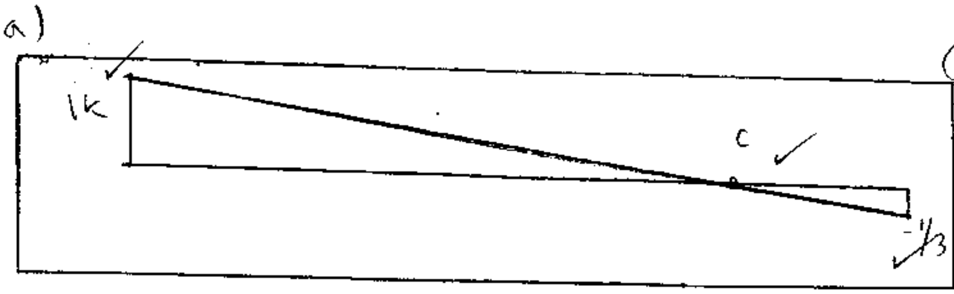
$\Sigma M_A = 0 = C_y(30)$

$C_y = 0$

$A_y = 1$

a)

IL for A_y



(0, 1) (30, 0)

$\frac{0-1}{30-0} = \frac{-1}{30}$

$1 = 0(1/30) + b$

$y = -\frac{1}{30}x + 1$

$y = -1/30(40) + 1$

$-1/3$

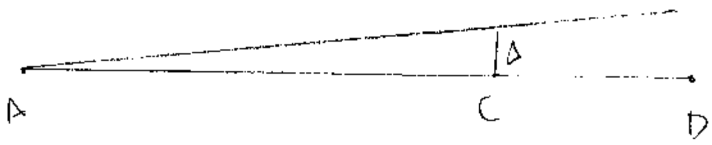
Unit load at D

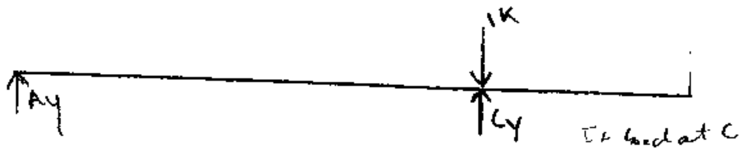
$\Sigma M_C = 0 = 1(10) + A_y(30)$

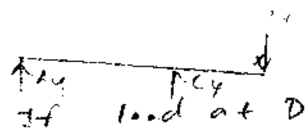
$A_y = -1/3$

Δ

b) release support at C







$\overset{+}{\curvearrowleft}\Sigma M_A = 0 = C_y(30) - 1(30)$

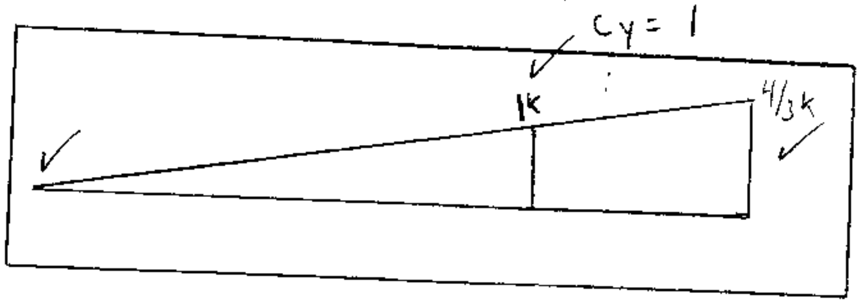
$C_y = 1$

$\Sigma M_A = 0 = C_y(30) - A_y(40)$

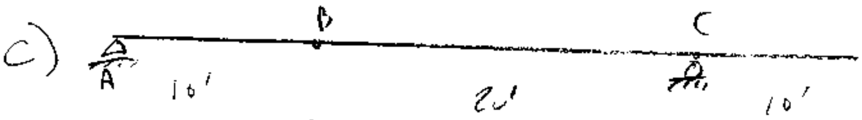
$C_y = 40/30$

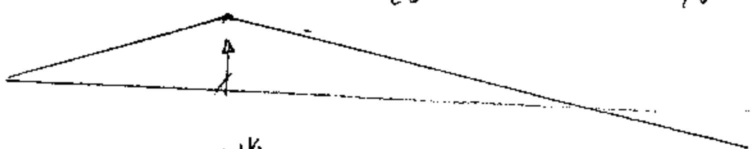
$C_y = 4/3$

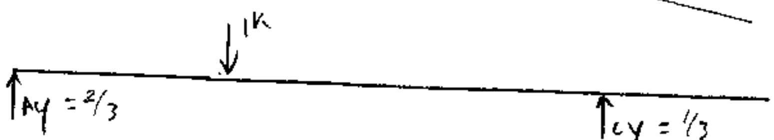
vertical rxn at C



c)







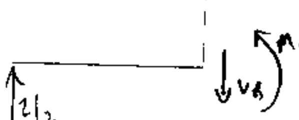
$\Sigma F_y = 0 = A_y + C_y - 1$

$A_y + C_y = 0$

$\overset{+}{\curvearrowleft}\Sigma M_A = 0 = 1(10) - C_y(30)$

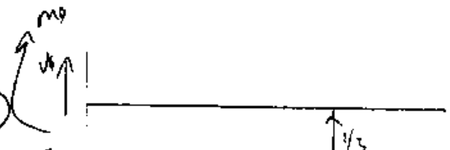
$C_y = 1/3$

$A_y = 2/3$



$\overset{+}{\curvearrowleft}\Sigma M_B = 0 = M_B - 2/3(10)$

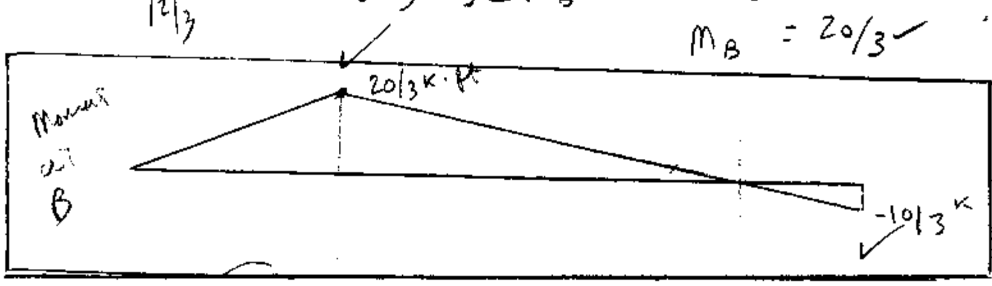
$M_B = 20/3$



$\Sigma M_B = 0 = M_B - 1/3(20)$

$M_B = 20/3$

Moment at B

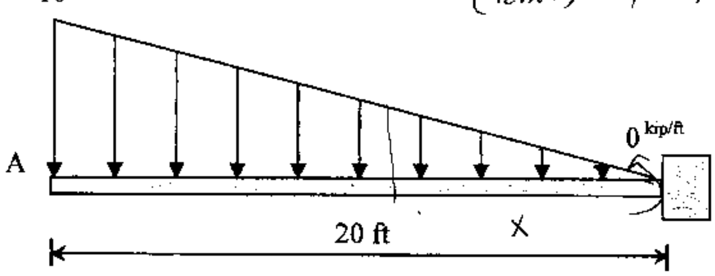


$y = -1/3(30) + 20/3$ $\quad (0, \frac{20}{3})\ (20, 0)$ $\quad \dfrac{\frac{20}{3} - 0}{0 - 20} = \dfrac{\frac{20}{3}}{20} = -1/3$

34

5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let E=29,000 ksi and I=4200 in^4. Your answer should be given in inches.

$.2025 = 4200\,in^4 \frac{1\,ft^4}{(12\,in^4)}$ 4,176,000 $\quad 29{,}000\frac{k}{in^2} \cdot \frac{(12\,in)^2}{1\,ft}$



$E = 29{,}000$ ksi

$I = 4200\ in^4$

$\frac{x}{h} = \frac{20}{10}$

$h(20) = x(10)$

$h = \frac{x}{2}$

$\Sigma F_y = 0 = -10(20)[.5] + B_y$

$B_y = 100^k$ ✓

$\Sigma M_B = 0 = M_B + 10[20][.5][2/3 \cdot 20]$

$M_B = -1333.33$ ✓

$\frac{10}{20} = \frac{h}{x}$

$10x = h20$

$\Sigma M_X = 0 = M_X + 1333.33 - 100(x) + [\frac{x}{2}][x][1/3\,x][.5]$

moment arm

$-M_x = 1333.33 - 100x + \frac{x^3}{12}$

$M_x = -\frac{x^3}{12} + 100x - 1333.33 + \cancel{C_1}$

Curvature $= \frac{M_x}{EI}$ ✓

$EI\,\theta(x) = \int M(x)$ ✓

$EI\,\theta(x) = \frac{-x^4}{48} + 50x^2 - 1333.33x + C_1$

B.C.

$\theta(x=0) = 0 = 0 + 0 - 0 + C_1$

$C_1 = 0$ ✓

$EI\,\theta(x) = \frac{-x^4}{48} + 50x^2 - 1333.33x$

$0 = -\frac{x^4}{48} + 50x^2 - 1333.33x$

$$EIy(x) = \frac{-x^5}{240} + \frac{50}{3}x^3 - 666.67x^2 + C_2$$

$$EI\,y(x=0) = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$$EI\,y(x) = \frac{-x^5}{240} + \frac{50}{3}x^3 - 666.67x^2$$

$$EIy(x=20) = \frac{-20^5}{240} + \frac{50}{3}(20^3) - 666.67(20^2)$$

$$\frac{\cancel{-13{,}333.33 + 133{,}333.33} - 266{,}666.67}{EI}$$

$$\boxed{y_{max} = -.31527\,in} \qquad \frac{-266{,}666.67}{4{,}176{,}000\,ft^4[.2025\,ft^4]}$$

Something wrong in conversion or math... -1

$$w = \tfrac{1}{2}x$$

$$V(x) = -\frac{x^2}{4} + C_1$$

$$V(x=0) = 100 = 0^2 + 0 + C_1$$

$$V(x) = \frac{-x^2}{4} + 100$$

$$M(x) = \frac{-x^3}{12} + 100x + C_2$$

$$M(x=0) = -1333.33$$

$$M(x) = \frac{-x^3}{12} + 100x - 1333.33$$

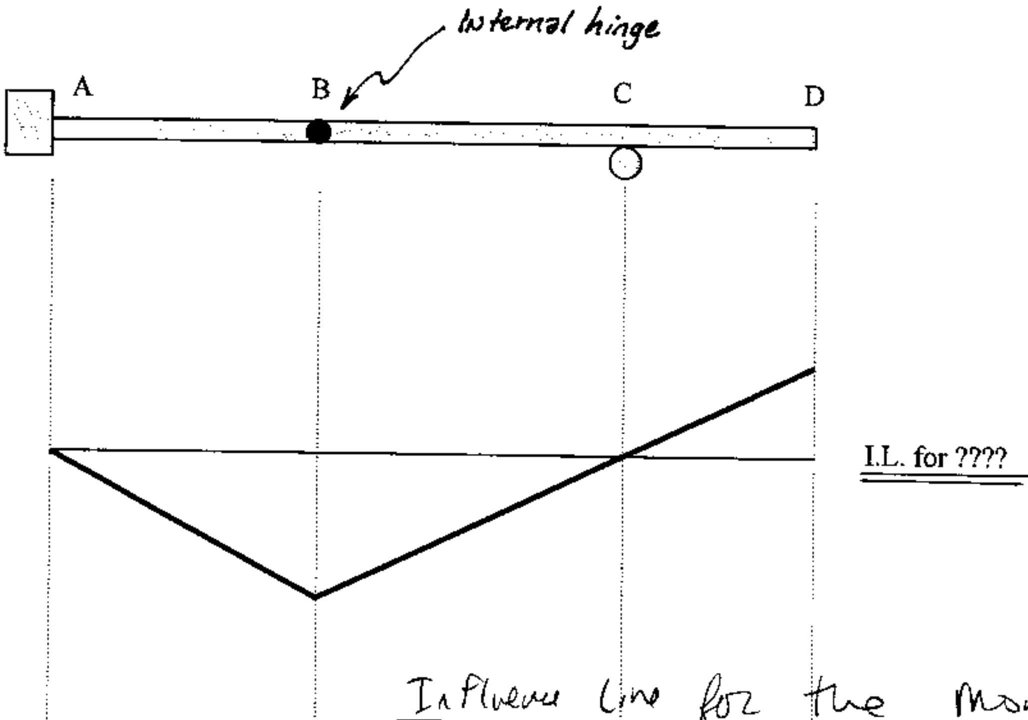
$$w = \tfrac{1}{2}x$$

$$V(x) = -\frac{x^2}{4} + C_1$$

$$M(x) = \frac{-x^3}{12} + C_1x + C_2$$

(5)

Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: "Influence Line for the Vertical Reaction at A".) Hint: The answer is not really "Influence Line for the Vertical Reaction at A". ☺

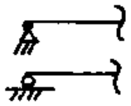
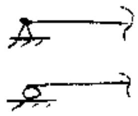
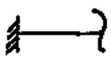
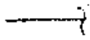
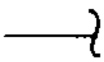
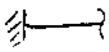
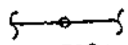
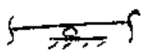


Influence Line for the Moment at A ✓

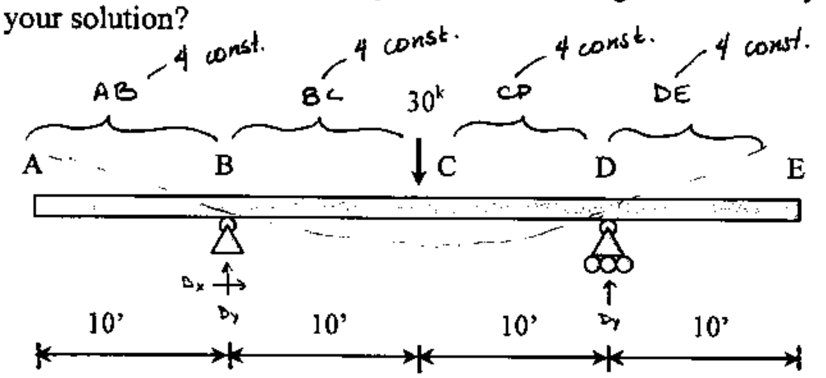
100/100 Excellent work!

CE 461 – Structural Analysis, Fall 2006 **Name:**_
EXAM No. 2

15 1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

REAL BEAM		**CONJUGATE BEAM**	
Type of Support	**Real Slope and Deflection**	**Conjugate Shear and Bending Moment**	**Type of Support**
Simple end support 	$\theta \neq 0$ $\Delta = 0$	$V \neq 0$ $M = 0$	
Fixed support (encastre) 	$\theta = 0$ $\Delta = 0$	$V = 0$ $M = 0$	 FREE END
Free end 	$\theta \neq 0$ $\Delta \neq 0$	$V \neq 0$ $M \neq 0$	 FIXED
Simple interior support 	$\theta \neq 0$ $\Delta = 0$	$V \neq 0$ $M = 0$	 INTERNAL HINGE
Internal hinge 	$\theta \neq 0$ $\Delta \neq 0$	$V \neq 0$ $M \neq 0$	 SIMPLE INTERIOR

0 2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?



A - E

$w(x) = 0$

$v(x) = \int w(x)\,dx = \int 0\,dx$

$= C_1$

$M(x) = \int v(x) = \int C_1$

$= C_1 x + C_2$

$\theta(x) = \int \frac{1}{EI} M(x)\,dx$

$= \frac{1}{EI} \int C_1 x + C_2$

$= \frac{1}{EI} \left(\frac{C_1 x^2}{2} + C_2 x + C_3 \right)$

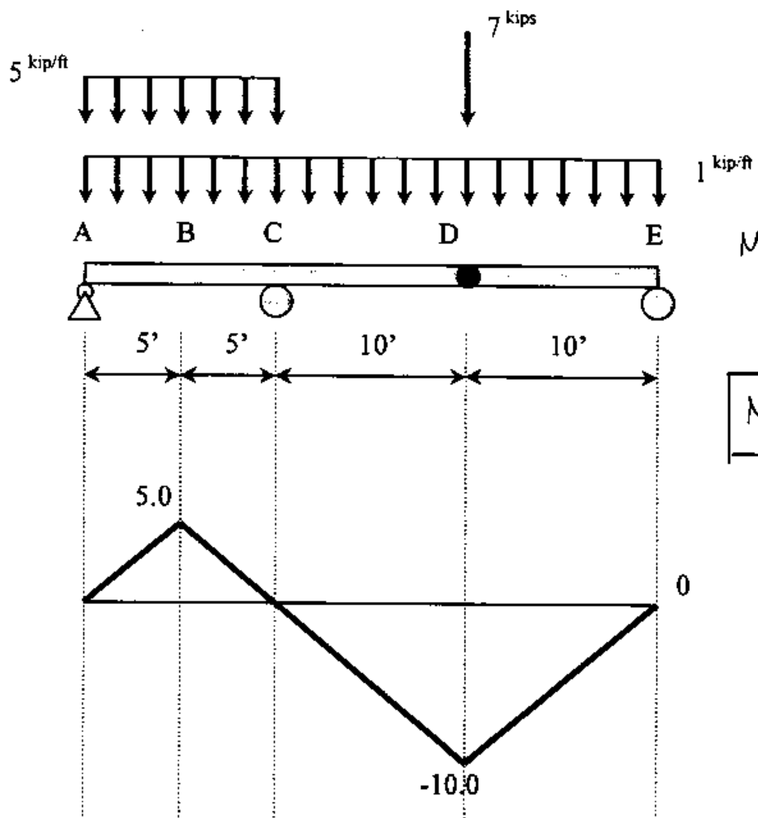
$y(x) = \int \theta(x)$

$= \int \frac{1}{EI} \left(\frac{C_1 x^2}{2} + C_2 x + C_3 \right)$

$= \frac{1}{EI} \left(\frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4 \right)$

$\boxed{\text{4 CONSTANTS}}$

10 3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the [redacted] moment at B generated by the loads shown.



25 + 125 − 100 − 70

$M_B = (1\ \text{k/ft})(5\ \text{k·ft/k} \times 5\ \text{ft}) + (5\ \text{k/ft})(5\ \text{k·ft/k} \times 5\ \text{ft})$
$- (1\ \text{k/ft})(10\ \text{k·ft/k} \times 10\ \text{ft}) - (7\ \text{k})(10\ \text{k·ft/k})$

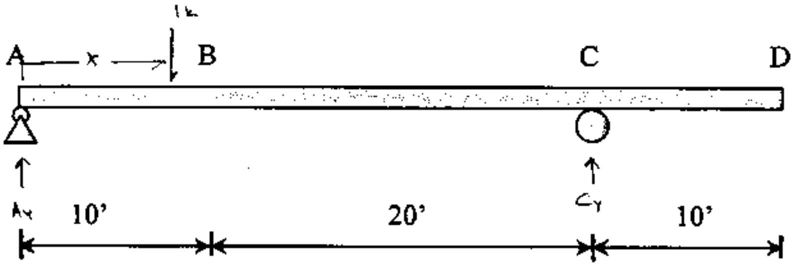
$\boxed{M_B = -20\ \text{k·ft}}$ ✓

Excellent work!

35 4. [35 pts] Given the following beam, draw the influence lines for:

- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

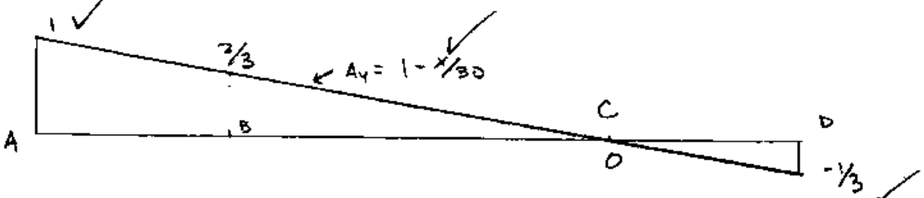
You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.



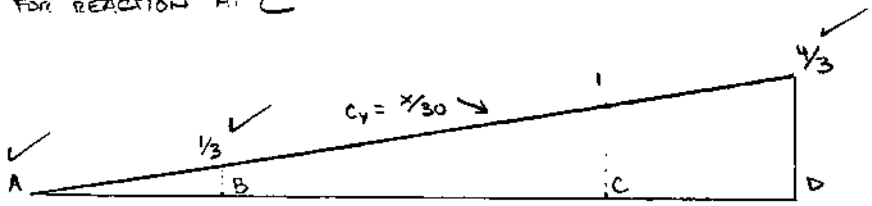
$\circlearrowleft + \Sigma M_A = 0 = -(1^k)(x) + C_y(30')$ $\therefore C_y = x/30$

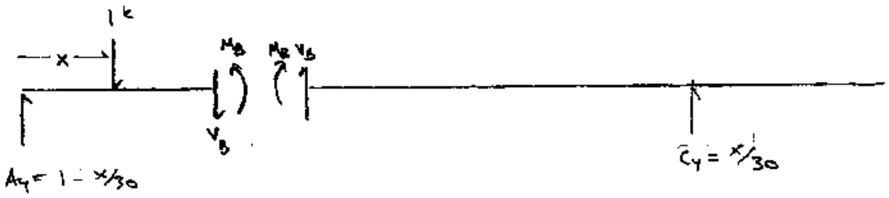
$\circlearrowleft + \Sigma M_C = 0 = (1^k)(30-x) - A_y(30)$ $\therefore A_y = 1 - x/30$

I.L. FOR REACTION AT A



I.L. FOR REACTION AT C

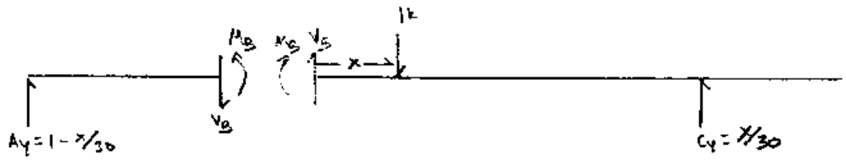




$$\circlearrowright + \Sigma M_{B\,(BD)} = 0 = -M_B + C_y\,(20\text{ ft})$$

$$M_B = 20\,C_y$$

$$M_B = \frac{2x}{3} \qquad 0' \le x \le 10'$$

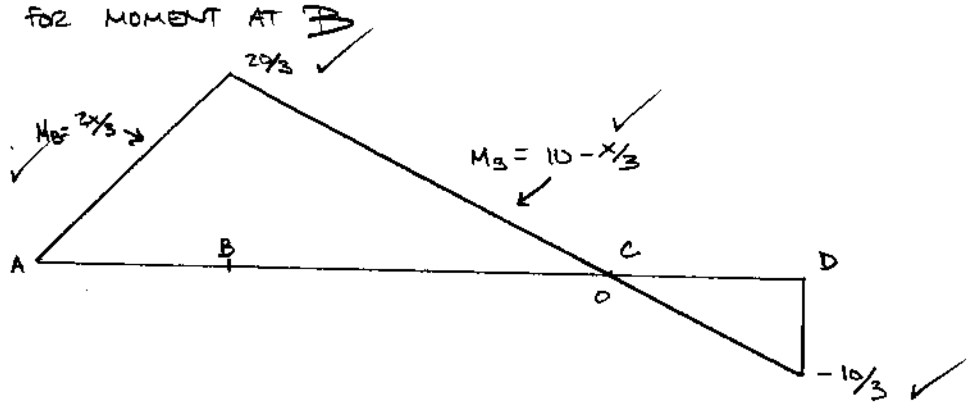


$$\circlearrowright + \Sigma M_{B\,(AB)} = 0 = M_B - (A_y)(10\text{ ft})$$

$$M_B = 10\,A_y$$

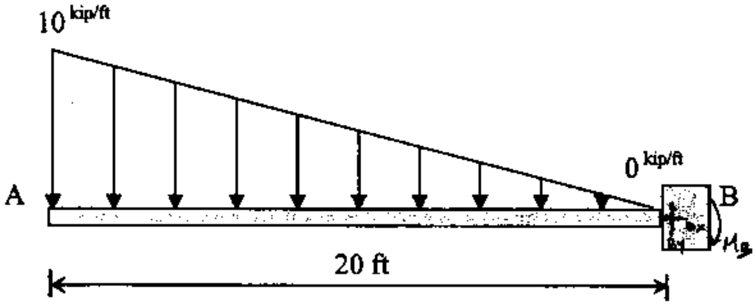
$$M_B = 10 - \frac{x}{3} \qquad 10' \le x \le 40'$$

I-L FOR MOMENT AT B



Great!

35 5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let E=29,000 ksi and I=4200 in^4. Your answer should be given in inches.



$\omega(x) = (10 - \frac{1}{2}x)$

$V(x) = \int -\omega(x)\,dx$

$= \int -(10 - \frac{1}{2}x)\,dx$

$= \int -10 + \frac{1}{2}x$

$V(x) = \frac{x^2}{4} - 10x + C_1$

$M(x) = \int V(x)\,dx$

$= \int (\frac{x^2}{4} - 10x + C_1)\,dx$

$M(x) = \frac{x^3}{12} - 5x^2 + C_1x + C_2$

$EI\,\theta(x) = \int M(x)\,dx$

$= \int (\frac{x^3}{12} - 5x^2 + C_1x + C_2)\,dx$

$= \frac{x^4}{48} - \frac{5x^3}{3} + \frac{C_1x^2}{2} + C_2x + C_3$

$\theta(x) = \frac{1}{EI}(\frac{x^4}{48} - \frac{5x^3}{3} + \frac{C_1x^2}{2} + C_2x + C_3)$

$y(x) = \int \theta(x)\,dx$

$= \frac{1}{EI}\int(\frac{x^4}{48} - \frac{5x^3}{3} + \frac{C_1x^2}{2} + C_2x + C_3)$

$y(x) = \frac{1}{EI}\left(\frac{x^5}{240} - \frac{5x^4}{12} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4\right)$

B.C.

$V(x=0)=0$

$0 = \frac{(0)^2}{4} - 10(0) + C_1 \quad \therefore C_1 = 0$

B.C.

$M(x=0)=0$

$0 = \frac{0^3}{12} - 5(0)^2 + \cancel{C_1}(0) + C_2 \quad \therefore C_2 = 0$

B.C.

$\theta(x=20)=0$

$0 = \frac{1}{EI}\left(\frac{20^4}{48} - \frac{5(20)^3}{3} + C_3\right)$

$0 = \frac{160000}{48} - \frac{40000}{3} + C_3$

$0 = \frac{160000}{48} - \frac{640000}{48} + C_3$

$\therefore C_3 = 10000$

B.C.

$y(x=20)=0$

$0 = \frac{1}{EI}\left(\frac{20^5}{240} - \frac{5(20)^4}{12} + 10000(20) + C_4\right)$

$0 = \frac{3200000}{240} - \frac{800000}{12} + 200000 + C_4$

$0 = \frac{3200000}{240} - \frac{16000000}{240} + \frac{48000000}{240} + C_4$

$\therefore C_4 = -\frac{35200000}{240}$

$$y(x) = \frac{1}{EI}\left(\frac{x^5}{240} - \frac{5x^4}{12} + 10000\,x - \frac{35200000}{240}\right)$$

$$Y_{MAX}(x=0) = \frac{1}{EI}\left(\frac{(0)^5}{240} - \frac{5(0)^4}{12} + 10000(0) - \frac{35200000}{240}\right)$$

$$E = \frac{29{,}000\ k}{in^2} \times \frac{144\ in^2}{1\ ft^2} = 4176000\ k/sf$$

$$I = 4200\,in^4 \times \left(\frac{1\,ft}{12\,in}\right)^4$$

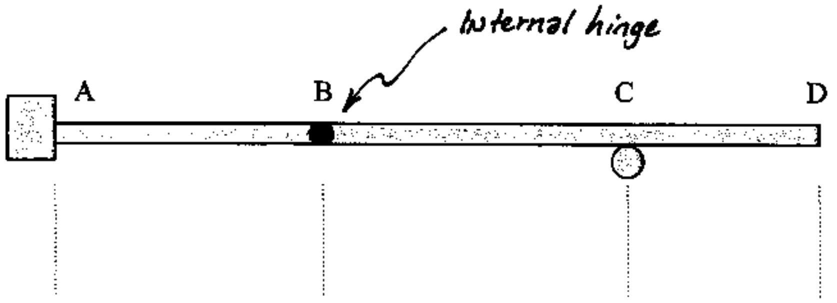
$$I = \frac{4200}{20736}$$

$$Y_{MAX} = -\frac{35200000}{240\,(4176000\ ksf)\left(\frac{4200}{20736}\right)}$$

203000000

$$\boxed{Y_{MAX} = -0.173\,ft \quad or \quad -2.08\,in}$$

(5) Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: "Influence Line for the Vertical Reaction at A".) Hint: The answer is not really "Influence Line for the Vertical Reaction at A". ☺



INFLUENCE LINE FOR THE MOMENT AT A ✓

105/100 WOW! perfect!!

CE 461 – Structural Analysis, Fall 2006 **Name**
EXAM No. 2

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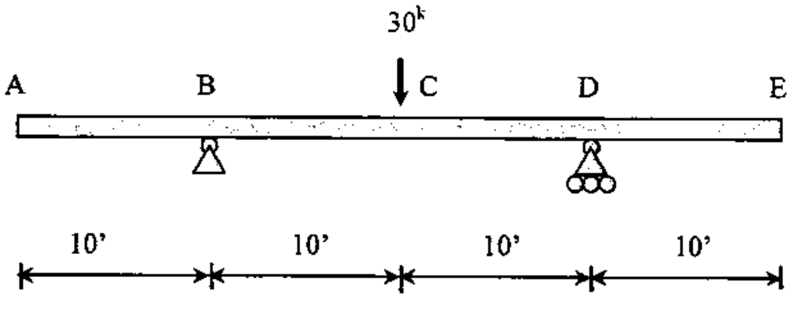
Simple end support	$\theta \neq 0$ $\Delta = 0$	$V \neq 0$ $M = 0$	Simple
Fixed support (encastre)	$\theta = 0$ $\Delta = 0$	$V = 0$ $M = 0$	Free
Free end	$\theta \neq 0$ $\Delta \neq 0$	$V \neq 0$ $M \neq 0$	Fixed
Simple interior support	$\theta \neq 0$ $\Delta = 0$	$V \neq 0$ $M = 0$	Hinge
Internal hinge	$\theta \neq 0$ $\Delta \neq 0$	$V \neq 0$ $M \neq 0$	Interior Support

5

2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?



$V = -\int w\,dx$

$M = \int V\,dx$

$\theta = \frac{1}{EI}\int M\,dx$

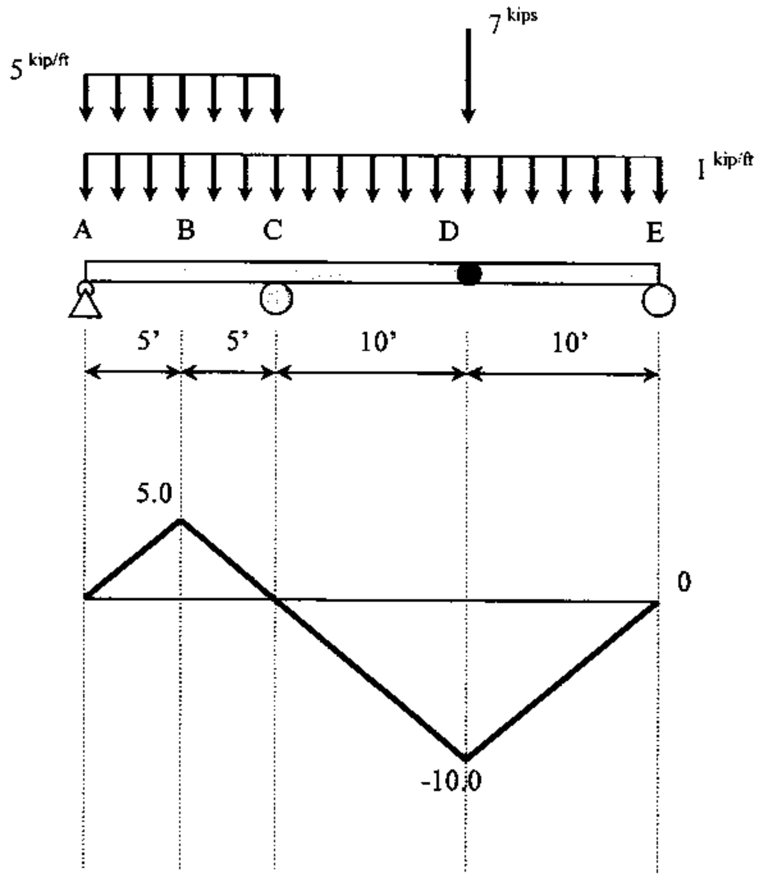
$v = \int \theta\,dx$

4 integrations per section ×
4 sections to analyze
<u>16 integration constants</u>

✓

10

3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the [redacted] moment at B generated by the loads shown.



$M_B = (\frac{1}{2}\cdot 10\cdot 5)\cdot 6\,\frac{kip}{ft} - (\frac{1}{2}20\cdot 10)\cdot$
$-(7\cdot 10)$

$\boxed{M_B = -20\ ft\cdot kip}$

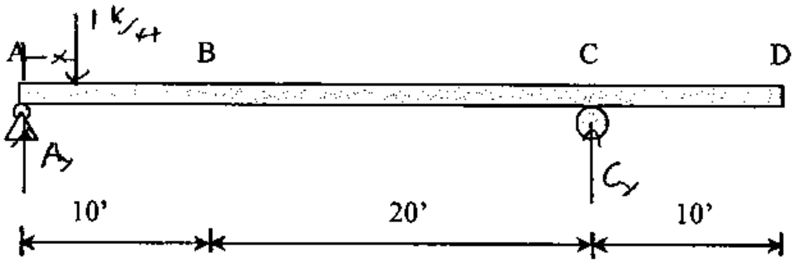
✓

Excellent!

35

4. [35 pts] Given the following beam, draw the influence lines for:
   - (10/35) Vertical reaction at A
   - (10/35) Vertical reaction at C, and
   - (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.



Use equilibrium method

Overall reaction:

$\Sigma M_A = 0$

$30 \cdot C_y - x = 0$

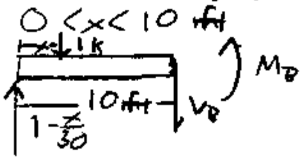
$C_y = \frac{x}{30}$

$\Sigma F_y = 0;\ A_y + C_y - 1 = 0$

$A_y = 1 - \frac{x}{30}$

Break beam, solve for moment at B

$0 < x < 10$ ft



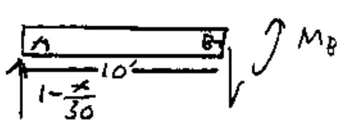
$\Sigma M_B = 0$

$-(1 - \frac{x}{30}) \cdot 10 + 1 \cdot (10 - x) + M_B = 0$

$M_B = 10 - \frac{x}{3} - 10 + x$

$M_B = \frac{2x}{3};\ 0 < x < 10 \qquad M(10^-) = \frac{20}{3} = 6.667$ ✓

$10 < x < 40$ ft



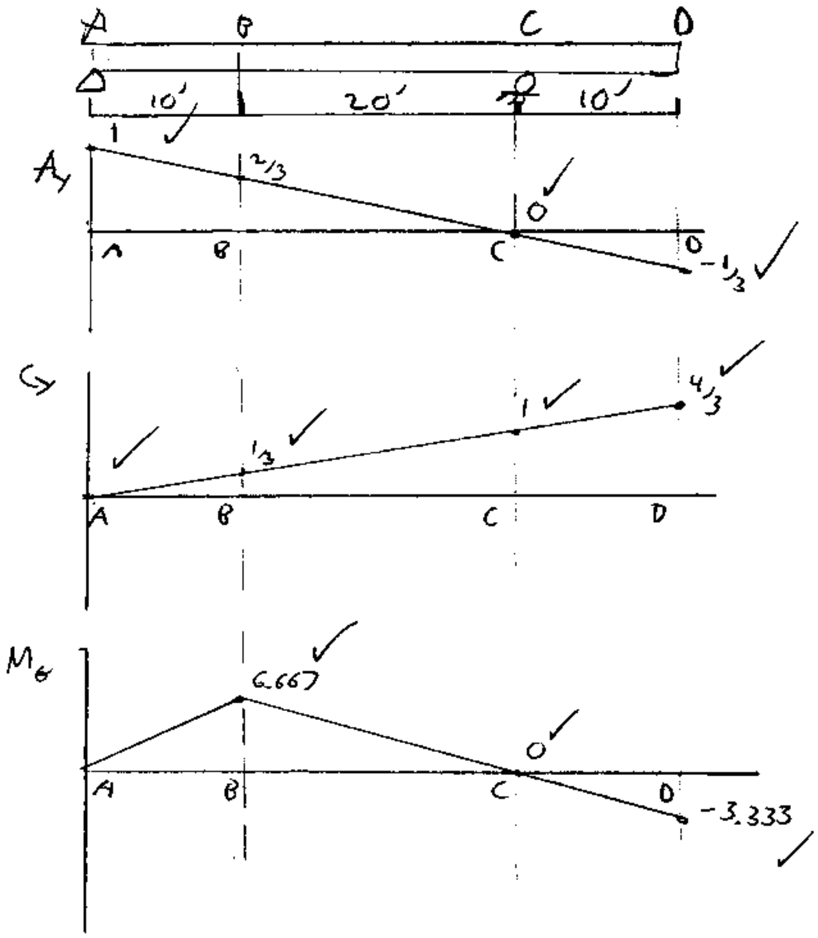
$\Sigma M_B = 0$

$-(1 - \frac{x}{30}) \cdot 10' + M_B = 0$

$M_B = 10 - \frac{10x}{30} = 10 - \frac{x}{3}$

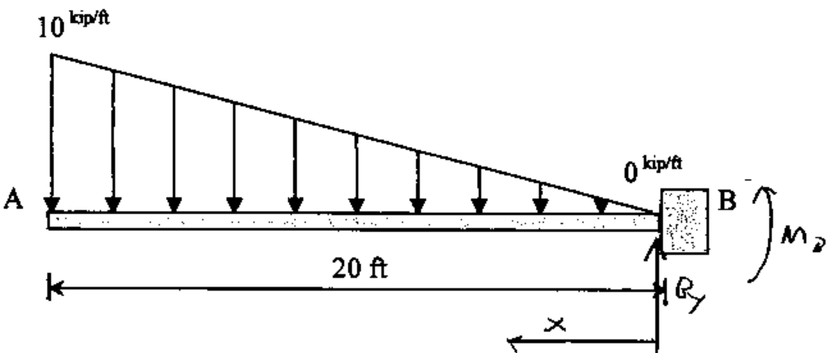
$M(10^+) = \frac{20}{3} = 6.667$ ✓

I.L on next page.


A
B
C
D
10'
20'
10'
1
A_y
2/3
0
-1/3
C_y
1/3
1
4/3
M_B
6.667
0
-3.333

Great!

35 5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let E=29,000 ksi and I=4200 in^4. Your answer should be given in inches.



Let $x=0$ at B, positive to the left

Overall rxn:

$\Sigma F_y = 0$

$-\frac{1}{2} \cdot 10 \cdot 20 + B_y = 0$

$\underline{B_y = 100 \text{ Kip}}$ ✓

$\Sigma M_B = 0$

$(\frac{1}{2} \cdot 10 \cdot 20) \cdot \frac{2}{3}(20) + M_B = 0$

$\underline{M_B = -1333.333 \text{ ft-kip}}$ ✓

$w(x) = \frac{1}{2}(x)$, in the defined coordinate system

$V(x) = -\int w(x)\,dx$

$V(x) = -\frac{1}{4}x^2 + C_1$

$V(20) = 0$

$C_1 = \frac{+20^2}{4} = +100$

$V(x) = -\frac{1}{4}x^2 + 100$ ✓

$M(x) = \int V(x)\,dx$

$M(x) = \frac{-1}{12}x^3 + 100x + C_2$

$M(0) = +1333.33$ kip, $\therefore$ $C_2 = +1333.33$

Check: $M(20) = \frac{-1}{12} \cdot 20^3 + 100 \cdot 20 \mp 1333.33 = 0$ ✓

$\theta = \frac{1}{EI}\int M(x)\,dx$

$\theta(x) = \frac{1}{EI}\left(\frac{-1}{48}x^4 + 50x^2 + 1333x + C_3\right)$

$\theta(0) = 0$, $C_3 = 0$

$v(x) = \int \theta(x)\,dx$

$v(x) = \frac{1}{EI}\left(\frac{-1}{240}x^5 + \frac{50}{3}x^3 - \frac{2000}{3}x^2 + C_4\right)$

$v(0) = 0$, $C_4 = 0$

(over)

$$v(x) = \frac{-1}{EI}\left(\frac{1}{240}x^5 - \frac{50}{3}x^3 + \frac{2000}{3}x^2\right)$$

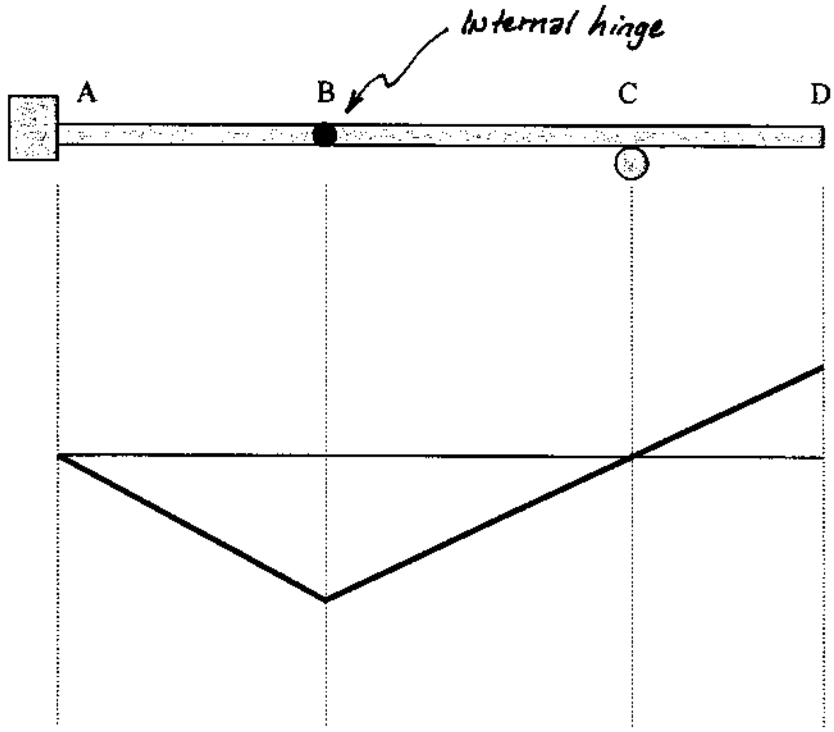
$EI = 29000\ ksi \cdot 4200\ in^4 = 1.218 \cdot 10^8\ kip \cdot in^2 \cdot \frac{1\ ft^2}{144\ in^2} = 8.458 \cdot 10^5\ kip \cdot ft^2$

Max deflection at end $(x=20)$

$$v(20) = \frac{-1}{8.458 \cdot 10^5}\left(\frac{1}{240} \cdot 20^5 - \frac{50}{3} \cdot 20^3 + \frac{2000}{3}x^2\right)$$

$$\boxed{v(20) = -0.1734\ ft = -2.08\ in\ \therefore \downarrow}$$

5 Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: "Influence Line for the Vertical Reaction at A".) Hint: The answer is not really "Influence Line for the Vertical Reaction at A". ☺



I.L. for ????

I.L for Moment at A

✓

81/100

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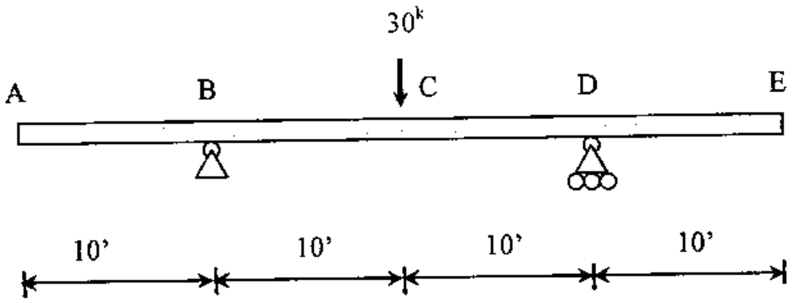
15 1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

REAL BEAM		**CONJUGATE BEAM**	
Type of Support	**Real Slope and Deflection**	**Conjugate Shear and Bending Moment**	**Type of Support**
Simple end support	$\theta \neq 0$ $\Delta = 0$	✓ $V \neq 0$ ✓ $M = 0$	Simple Ends ✓
Fixed support (encastre)	$\theta = 0$ $\Delta = 0$	✓ $V = 0$ ✓ $M = 0$	Free ✓
Free end	$\theta \neq 0$ $\Delta \neq 0$	✓ $V \neq 0$ ✓ $M \neq 0$	Fixed ✓
Simple interior support	$\theta \neq 0$ $\Delta = 0$	✓ $V \neq 0$ ✓ $M = 0$	Internal hinge ✓
Internal hinge	$\theta \neq 0$ $\Delta \neq 0$	✓ $V \neq 0$ ✓ $M \neq 0$	Simple interior Support ✓

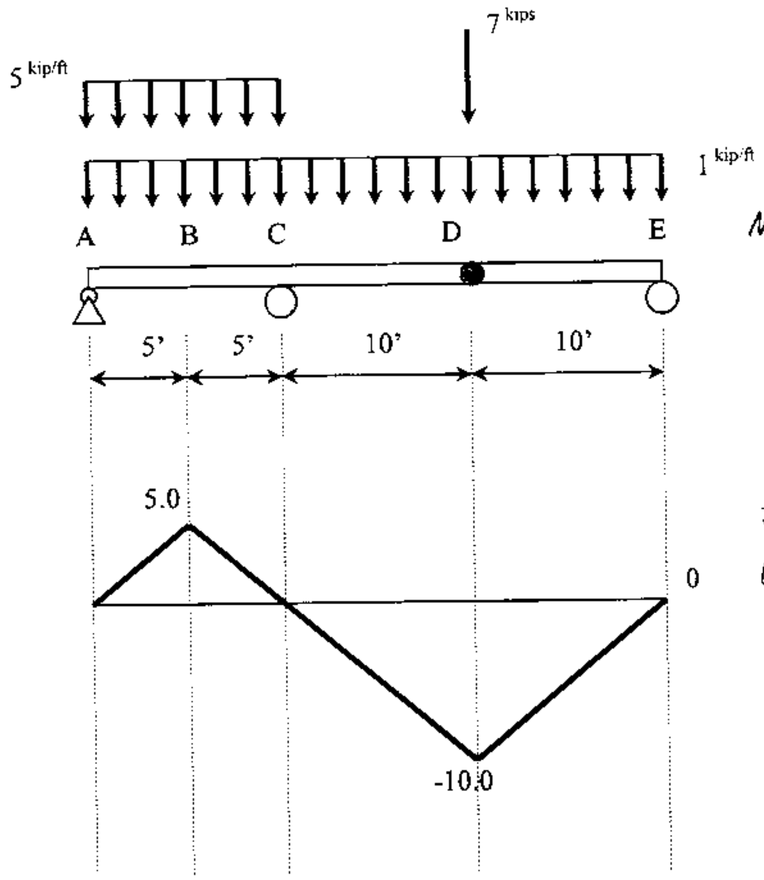
5 2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?



From A to B 4 const
From B to C 4 const
From C to D 4 const
From D to E 4 const

$\boxed{\therefore 16 \text{ const of Intergration}}$ ✓

3 3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the [redacted] moment at B generated by the loads shown.



$M_B = (5^{k/ft})(10')(2)(\frac{1}{2})(5)(5) + (1^{k/ft})(10')(2)(\frac{1}{2})(5)(5)$

$+ (1^{k})(20')(2)(\frac{1}{2})(-10)(10) + 7(-10)$ ✓

$M_B = 1250 + 250 - 2000 - 70$

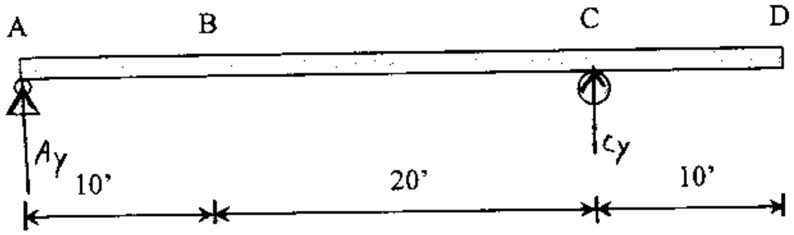
$\boxed{M_B = -570^{k\cdot ft}}$ × -7

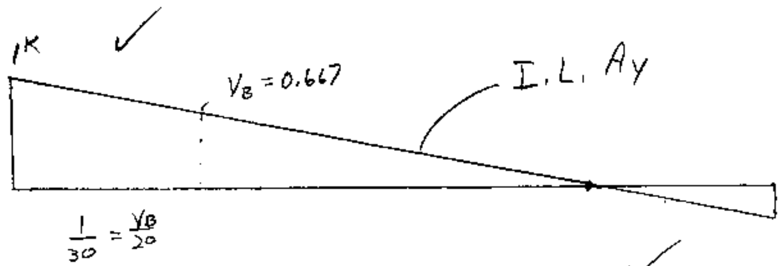
28

4. [35 pts] Given the following beam, draw the influence lines for:

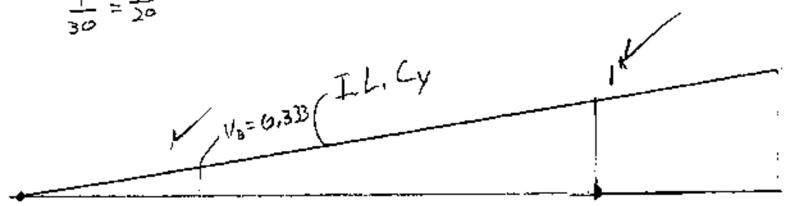
- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

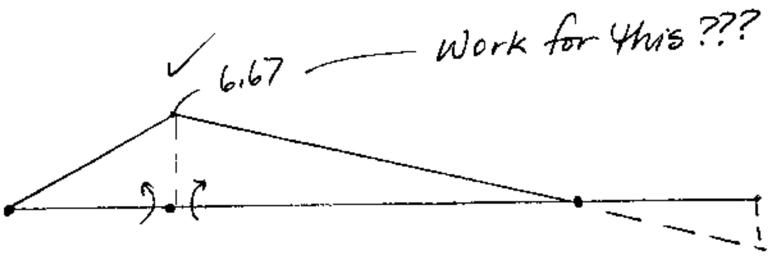




Release the reaction @ A.



Release the reaction @ C.



Release the reaction @ B.

−7

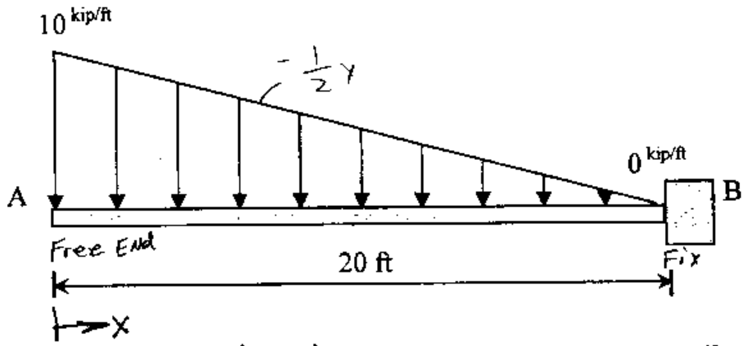
30 5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let E=29,000 ksi and I=4200 in^4. Your answer should be given in inches.

$V(0) = 0$

$M(0) = 0$

$\theta(20) = 0$

$y(20) = 0$



$w(x) = \frac{1}{2}(10)(20-x)$

$w(x) = -100 + 5x$

$V(x) = \int w(x)\,dx$

$V(x) = \int(-100 + 5x)\,dx$

$V(x) = -100x + \frac{5}{2}x^2 + C_1$

B.C. 1 $V(0) = 0$

$-100(0) + \frac{5}{2}(0)^2 + C_1 = 0$

$C_1 = 0$

$\therefore V(x) = -100x + \frac{5}{2}x^2$

$M(x) = \int V(x)\,dx$

$M(x) = \int(-100x + \frac{5}{2}x^2)\,dx$

$M(x) = -50x^2 + \frac{5}{6}x^3 + C_2$

B.C. 2 $M(0) = 0$

$-50(0^2) + \frac{5}{6}(0^3) + C_2 = 0$

$C_2 = 0$

$\therefore M(x) = -50x^2 + \frac{5}{6}x^3$

$EI\theta(x) = \int M(x)\,dx$

$\theta(x) = \frac{1}{EI}\int(-50x^2 + \frac{5}{6}x^3)\,dx$

$\theta(x) = \frac{1}{EI}(-\frac{50}{3}x^3 + \frac{5}{24}x^4 + C_3)$

B.C. 3 $\theta(20) = 0$

$\frac{1}{EI}(-\frac{50}{3}(20)^3 + \frac{5}{24}(20^4)) + C_3 = 0$

$C_3 = +100{,}000$

$\therefore \theta(x) = \frac{1}{EI}(-\frac{50}{3}x^3 + \frac{5}{24}x^4 + 100{,}000)$

$y(x) = \frac{1}{EI}\int \theta x\,dx$

$y(x) = \frac{1}{EI}\int(-\frac{50}{3}x^3 + \frac{5}{24}x^4 + 100{,}000)$

$y(x) = \frac{1}{EI}(-\frac{50}{12}x^4 + \frac{5}{120}x^5 + 100{,}000x + C_4)$

B.C. 4 $y(20) = 0$

$\frac{1}{EI}(-\frac{50}{12}(20^4) + \frac{5}{120}(20^5) + 100{,}000(20) + C_4 = 0$

$C_4 = -1{,}466{,}666.7$

$\therefore y(x) = \frac{1}{EI}(\frac{50}{12}x^4 - \frac{5}{120}x^5 - 100{,}000x - 1{,}466{,}66$

Next Page

~~y_{max} @ $\theta = 0$, or in this case at the Free End~~

~~y_{max} @ $x = 0$~~

~~$\therefore\ y_{max} = \frac{1}{EI}\left(-\frac{20}{12}0^4 + \frac{5}{120}(0^5) + 100{,}000(0) - 1{,}466{,}666.7\right)$~~

~~$y_{max} = \frac{-1466{,}666.7(12^3)}{(29000)(4200)}$~~

~~$y_{max} = -20.8''$ to Big !!~~

~~Not sure what I did wrong?~~

$w = 10 - \frac{1}{2}x$

$w(x) = -\frac{1}{2}x \times$ -5

$V(x) = \int -\frac{1}{2}x\,dx$

$V(x) = -\frac{1}{4}x^2 + C_1$

$C_1 = 0$ ✓

$V(x) = -\frac{1}{4}x^2$

$M(x) = \int -\frac{1}{4}x^2\,dx$

$M(x) = -\frac{1}{6}x^3 + C_2$

$C_2 = 0$ ✓

$M(x) = -\frac{1}{6}(x^3)$

$EI\theta = \int -\frac{1}{6}x^3\,dx$

$\theta(x) = \frac{1}{EI}\left(-\frac{1}{24}x^4 + C_3\right)$

$\theta(20) = 0$ ✓ $0 = -\frac{1}{24}(20^4) + C_3 \quad C_3 = 6667$

$\theta(x) = \frac{1}{EI}\left(-\frac{5}{24}x^4 + 6{,}667\right)$

$y(x) = \frac{1}{EI}\int -\frac{1}{24}x^4 + 6{,}667)\,dx$

$y(x) = \frac{1}{EI}\left(-\frac{1}{120}x^5 + 6{,}667 + C_4\right.$ ✓

$y(20) = 0 = \frac{-1}{120}(20^5) + 6667(20) + C_4$

$C_4 = -106{,}673$

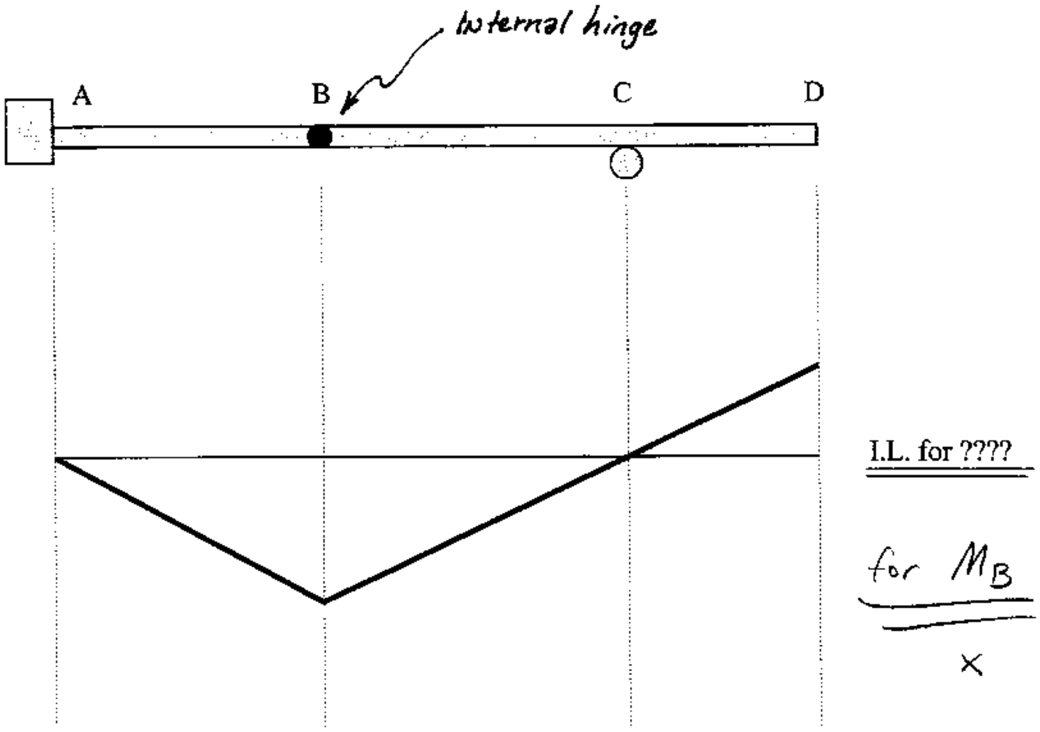
$y(x) = \frac{1}{EI}\left(\frac{-5}{120}(x^5) + 6667(x) + 106{,}673\right)$

$y(0) = \frac{106{,}673(12^3)}{29000(4200)} = 1.51''$ ✓

$\boxed{\therefore\ y_{max} = 1.51''}$ x

Load function incorrect

0 Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: "Influence Line for the Vertical Reaction at A".) Hint: The answer is not really "Influence Line for the Vertical Reaction at A". ☺



98.5/100 WooHoo!

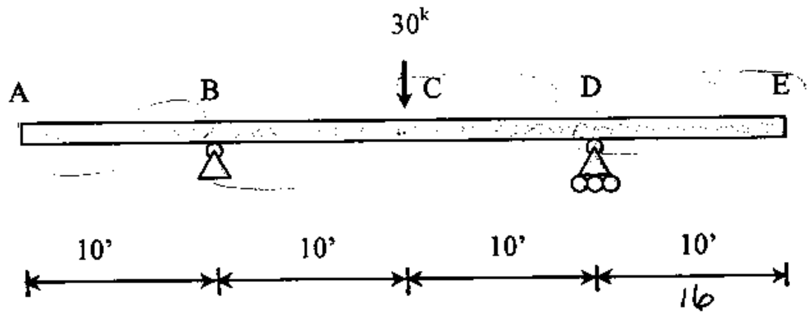
CE 461 – Structural Analysis, Fall 2006 **Name:**_
EXAM No. 2 10/9/06

14 1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

REAL BEAM		**CONJUGATE BEAM**	
Type of Support	**Real Slope and Deflection**	**Conjugate Shear and Bending Moment**	**Type of Support**
Simple end support	$\theta \neq 0$ $\Delta = 0$	✓ $V \neq 0$ ✓ $M = 0$	Simple end support ✓
Fixed support (encastre)	$\theta = 0$ $\Delta = 0$	✓ $V = 0$ ✓ $M = 0$	Free end ✓
Free end	$\theta \neq 0$ $\Delta \neq 0$	✓ $V \neq 0$ ✓ $M \neq 0$	Fixed support (encastre) ✓
Simple interior support	$\theta \neq 0$ $\Delta = 0$	✓ $V \neq 0$ ✓ $M = 0$	Internal hinge ✓
Internal hinge	$\theta \neq 0$ $\Delta = 0$ -0.5	✓ $V \neq 0$ ✗ $M = 0$ -0.5	Simple interior support ✓

-1

3 2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?



A-B w

$V = \int w = wx + c_1$

$M = \frac{wx^2}{2} + c_1 x + c_2$

$\theta = \frac{wx^3}{6} + \frac{c_1 x^2}{2} + c_2 x + c_3$

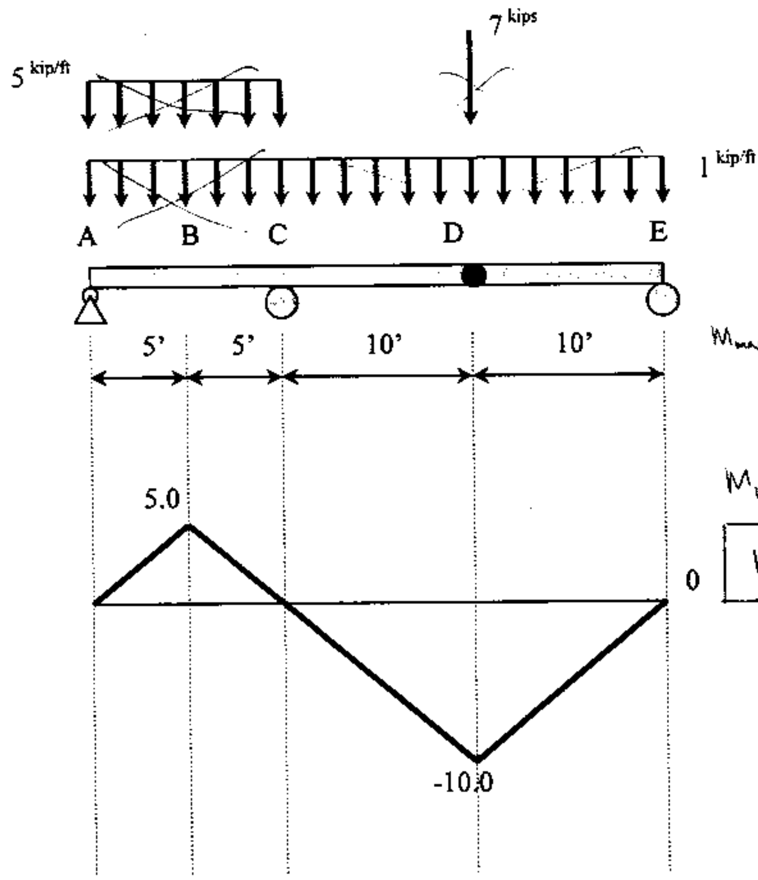
$Y = \quad c_1 x^3 \; c_2 x^2 + c_3 x + c_4$

$4 + 4 + 4 + 4 = 12$

One would have to account for ~~12~~ 16 constants of integration in the solution

Four distinct regions, ea. w/ 4 constants -2

10 3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the [redacted]) moment at B generated by the loads shown.



I.L. M_B -

$M_{max} = \frac{1}{2}(5)(10)(1) + \frac{1}{2}(5)(10)(5) - \frac{1}{2}(10)(20)(1) - 7(10)$

$M_{max} = 25 + 125 - 100 - 70$

$\boxed{M_{max} = -20 \text{ kip}\cdot\text{ft}}$ ✓

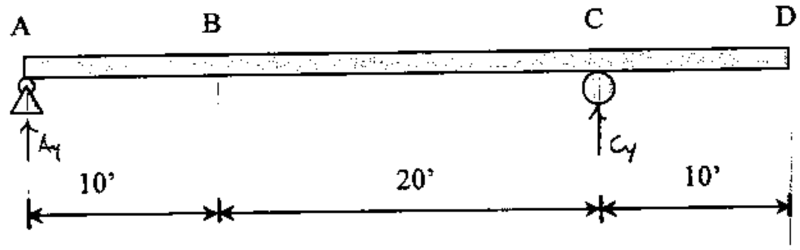
good!

Excellent!

35

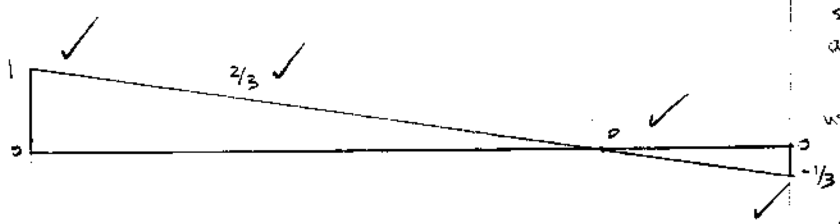
4. [35 pts] Given the following beam, draw the influence lines for:
   - (10/35) Vertical reaction at A
   - (10/35) Vertical reaction at C, and
   - (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.



A_y

I.L. for A_y



M.B. method: small deflection of released structure at A.
apply 1 k load ~~at~~ A.

using similar triangles to find ordinates at B & D

$\frac{1}{30} = \frac{B}{20}$ $B = 2/3$

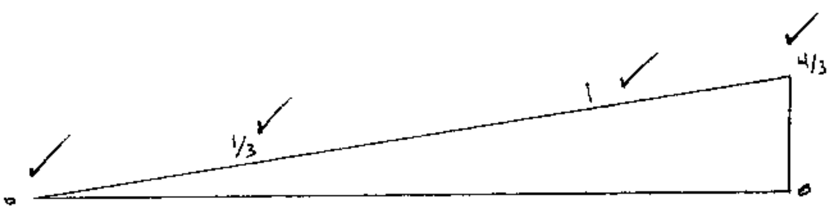
$y = mx + b$

$y = -\frac{1}{30}x + 1$

$y(40) = -\frac{40}{30} + 1 = -1/3$

C_y

I.L. for C_y



M.B. method: small deflection of released structure at C. apply 1 k load at C to find ordinate @ C.

Similar triangles to find other ordinates

$\frac{1}{30} = \frac{B}{10}$ $B = 1/3$

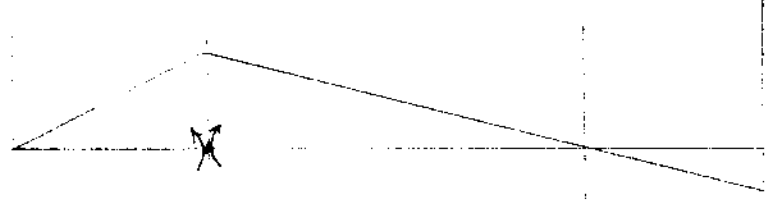
$\frac{1}{30} = \frac{C}{40}$ $C = 4/3$

$Y = mx + b$

$Y = \frac{\frac{4}{3}}{40}x + 0$

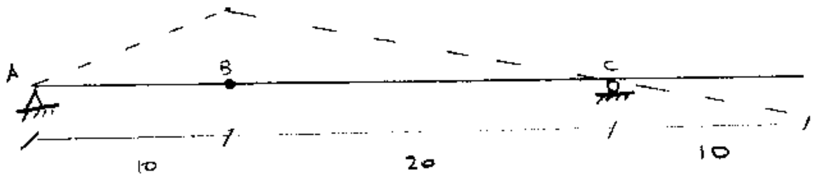
$Y = \frac{1}{30}x$

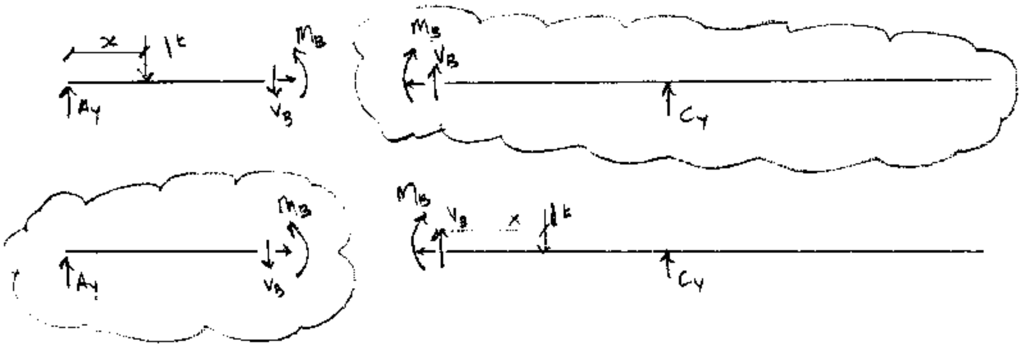
Intuitive I.L. for M_B



M.B. - insert an internal pin at B & apply small rotation keeping exterior constraints rigid.

Find ordinates of I.L.





$\Sigma M_B = 0 = M_B - C_y(20')$

$M_B = C_y(20)$

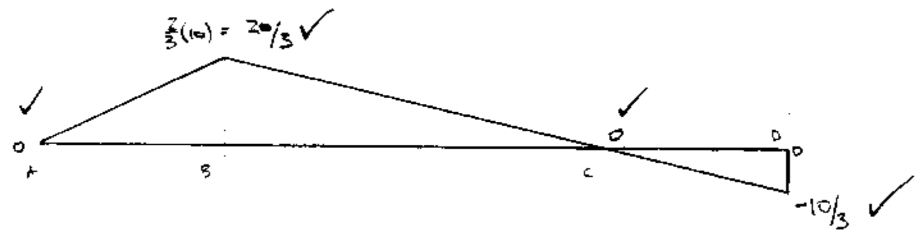
for $0' \le x \le 10'$

$\Sigma M_B = 0 = M_B - A_y(10)$

$M_B = 10A_y \qquad 10' \le x \le 40'$

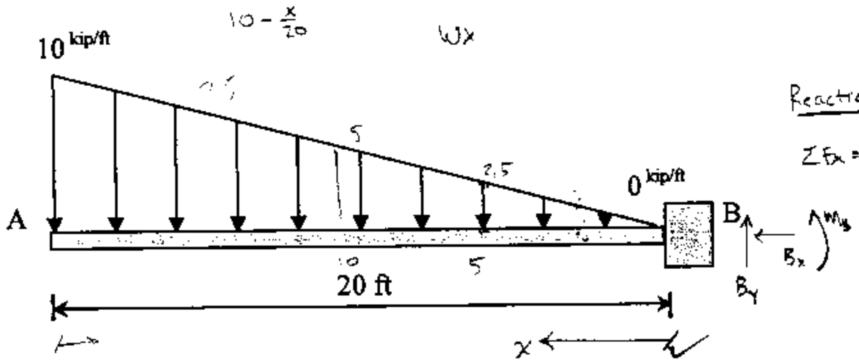
$$M_B = \begin{cases} C_y(20) = \frac{x}{30}(20) = \frac{2}{3}x & 0' \le x \le 10' \\ A_y(10) = \left(\frac{-x}{30} + 1\right)(10) = -\frac{1}{3}x + 10 & 10' \le x \le 40' \end{cases}$$

I.L. for M_B



30 +1.5 = 31.5 (See note on last page)

5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let E=29,000 ksi and I=4200 in^4. Your answer should be given in inches.



$w = \frac{1}{2}x\left(\frac{1}{2}x\right) = .25x^2$

Reactions

$\Sigma F_x = 0 = B_x$

$\Sigma F_y = 0 = -\frac{1}{2}(10)(20) + B_y$

$B_y = 100^k$ ✓

$\Sigma M_B = 0 = M_B + \frac{1}{2}(10)(20)(20 \cdot \frac{2}{3})$

$M_B = -1333.33$ kip.ft ✓

$M_{D} = \frac{wx(\frac{2}{3}L)}{2} = \frac{wxL}{3}$

$EI\theta = \int M dx = \int +1333.33\, dx = -1333.33x + C_1$

B.C. $\theta(x=0) : 0 = +1333.33(0) + C_1$

$\therefore C_1 = 0$

$\theta = \frac{1}{EI}(+1333.33x)$

$y = \int \theta dx = \frac{1}{EI}\int +1333.33x\, dx = \frac{1}{EI}\left(\frac{+1333.33}{2}x^2 - C_2\right)$

B.C. $y(x=0) = 0 = \frac{1333.33(0)^2}{2} + C_2$

$\therefore C_2 = 0$

$y = \frac{1}{EI}\left(\frac{1333.33}{2}x^2\right)$

max deflection location will be at the end of the beam (free end)

$y = \frac{1}{EI}\left(\frac{1333.33}{2}(20)^2\right) = \frac{266666\ ft^3 \cdot k}{EI} = \frac{266,666\ k \cdot ft^3}{29,000 \frac{k}{in^2}\ (4200\ in^4)}$

$\underline{\underline{or}}$

$\times$ $w = .25x^2$? But the load is a straight line, not a parabola $w = \frac{1}{2}x$ (-5)

D

$V = \int -w\,dx = \int -.25x^2\,dx = \frac{-.25}{3}x^3 + C_1$

BC $V(x=0) = 100^k$

$V(x=0) = 100 = \frac{-.25}{3}(0)^3 + C_1$

you included here..? typo?

$\therefore C_1 = 0$ $\times$ $C_1 = 100$

$M = \int V\,dx = \int \frac{-.25}{3}x^3 + 100\,dx = \frac{-.25x^4}{12} + 100x + C_2$

B.C. $M(x=0) = -1333.33$

$-1333.33 = \frac{-.25(0)^4}{12} + 100(0) + C_2$

$\therefore C_2 = -1333.33$ ✓

$EI\theta = \int M\,dx = \int \frac{-.25x^4}{12} + 100x + -1333.33\,dx$

$= \frac{-.25}{60}x^5 + \frac{100}{2}x^2 + -1333.33x + C_3$

B.C. $\theta(x=0) = 0$

$\therefore C_3 = 0$ ✓

$\theta = \frac{1}{EI}\left(\frac{-.25}{60}x^5 + 50x^2 + -1333.33x\right)$

$Y = \int \theta\,dx = \frac{1}{EI}\int \frac{-.25x^5}{60} + 50x^2 + -1333.33x\,dx$

$= \frac{1}{EI}\left(\frac{-.25}{360}x^6 + \frac{50}{3}x^3 - \frac{1333.33x^2}{2} + C_4\right)$

B.C. $Y(x=0) = 0$

$\therefore C_4 = 0$ ✓

$Y = \frac{1}{EI}\left(\frac{-.25x^6}{360} + \frac{50}{3}x^3 - \frac{1333.33}{2}x^2\right)$

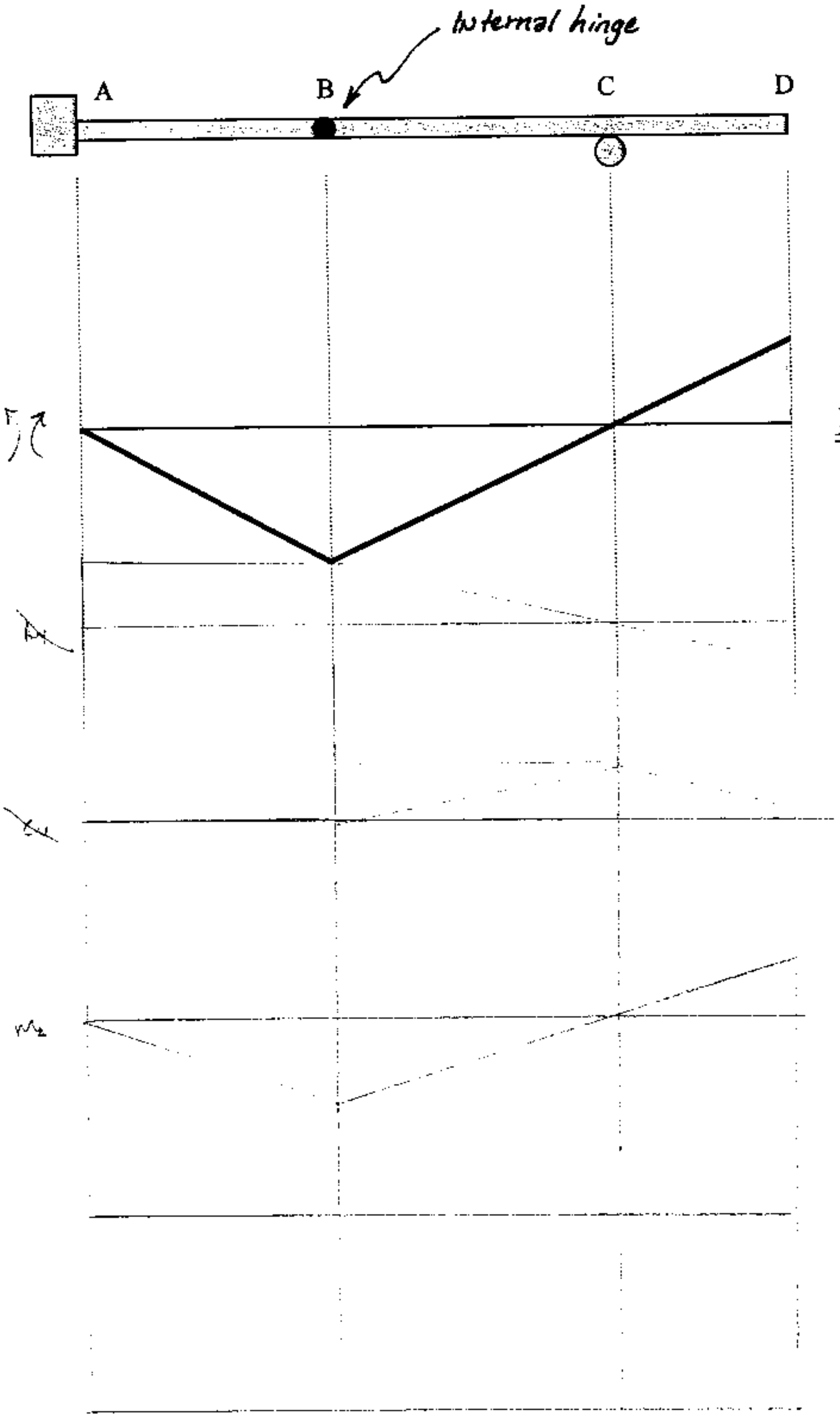
max deflection occurs at the free end of the beam $(x=20)$

$Y(x=20) = \frac{1}{EI}(-177{,}777.11) = \frac{-177777.11\ k \cdot ft^3}{29{,}000\ \frac{k}{in^2}\ (4{,}200\ in^4)} \cdot \frac{1\ ft^2}{144\ in^2} = -0.210\ ft$

$Y(x=20') = -.21\ ft \cdot \frac{12\ in}{ft} = -2.52$

$\boxed{Y_{@x=20'} = -2.52\ in.}$

5 Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: "Influence Line for the Vertical Reaction at A".) Hint: The answer is not really "Influence Line for the Vertical Reaction at A". ☺



I.L. for ???? M_A

Problem 5
Exam 2
1/9/06

②

W

$V = \int W\,dx = -Wx + c_1$

B.C. $V(x=0) = 100^k$

$V(x=0) = 100 = -W(0) + c_1$

$\therefore c_1 = 100$

$M = \int V\,dx = \int -Wx + 100\,dx = -\frac{W}{2}x^2 + 100x + C_2$

B.C. $M(x=0) = -1333.33$

$-1333.33 = -\frac{W}{2}(0)^2 + 100(0) + C_2$

$\therefore C_2 = -1333.33$

$EI\theta = \int M\,dx = \int -\frac{Wx^2}{2} + 100x - 1333.33\,dx$

$= -\frac{Wx^3}{6} + \frac{100x^2}{2} - 1333.33x + C_3$

B.C. $\theta(x=0) = 0$

$\therefore C_3 = 0$

$\theta = \frac{1}{EI}\left(\frac{-Wx^3}{6} + 50x^2 - 1333.33x\right)$

$Y = \int \theta\,dx = \frac{1}{EI}\int \frac{-Wx^3}{6} + 50x^2 - 1333.33x\,dx$

$= \frac{1}{EI}\left(\frac{-Wx^4}{24} + \frac{50x^3}{3} - \frac{1333.33x^2}{2} + C_4\right)$

B.C. $Y(x=0) = 0$

$\therefore C_4 = 0$

$Y = \frac{1}{EI}\left(\frac{-Wx^4}{24} + \frac{50x^3}{3} - \frac{1333.33x^2}{2}\right)$

if $W = \frac{1}{2}x$, then

$Y = \frac{1}{EI}\left(\frac{-x^5}{48} + \frac{50x^3}{3} - \frac{1333.33x^2}{2}\right)$

$Y(x=20) = \frac{1}{EI}\left(\frac{-(20)^5}{48} + \frac{50}{3}(20)^3 - \frac{1333.33}{2}(20)^2\right)$

$= \frac{-199999.33}{EI} = \frac{-199999.33}{(29000)(4200)\left(\frac{1}{144}\right)} = -0.2365\text{ ft} \times \frac{12''}{1'} = -2.838''$

$\boxed{Y(x=20') = -2.838''}$ ✓

This page has the correct solution... however, I can't give you full credit because of the dual solutions. You correctly ID'ed the problem, though, & I'll reduce the points deducted b/c of that. +45

I have worked this problem out 2 different ways using 2 different distributed load equations for w, b/c I cannot convince myself of the correct one. However, the two equations that I have arrived at for the deflection are the same except for the first term which is affected by the loading equation for W. Both answers seem reasonable & are fairly close to each other which makes it harder to distinguish the proper solution. I tried to work it out as far as possible w/out including "w" in the second solution as to minimize error as much as possible.