1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

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<td>$\theta \neq 0 \times$</td>
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<td></td>
</tr>
</tbody>
</table>

- 83.5/100
2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?

![Diagram of a beam with loads and sections labeled A through E.]

3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the moment at B generated by the loads shown.

![Influence line diagram for moment at B.]

\[
M_{\text{B}} = 50 + 2 \cdot k - 70 = 20
\]
4. [35 pts] Given the following beam, draw the influence lines for:
- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

---

CE 461–Exam No. 2, Fall 2006
CRB

True, but your origin for Ay started at 0, & this one starts at 10, so you must adjust. -5
\[ M_B = \begin{cases} \frac{20x}{30} & 0 \leq x \leq 10 \\ 10 - \frac{10x}{30} & 10 < x \leq 40 \end{cases} \]
5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let $E=29,000$ ksi and $I=4200$ in$^4$. Your answer should be given in inches.

![Diagram of beam with loads and deflections]

\[ M = 100x - 1,333.33 - \frac{1}{2} \left( x \right) \left( \frac{1}{2} x \right) \]

\[ M = 100x - 1,333.33 - \frac{1}{2} \left( \frac{1}{2} x \right) \left( \frac{1}{2} x \right) \]

\[ \theta = \int_0^x \frac{dM}{E} = \int_0^x \left( 50x^2 - 1,333.33x + C_1 \right) \frac{dx}{E_x} \]

\[ \gamma = \int_0^x \frac{d\theta}{dx} = \int_0^x \left( 16.67x^3 - 666.66x^2 + C_1x + C_2 \right) \frac{dx}{E_x} \]

\[ \theta(0) = 0 \Rightarrow C_1 = 0 \]

\[ \gamma(0) = 0 \Rightarrow C_2 = 0 \]

\[ \text{Max. Def.} \]

\[ \gamma = \left( 16.67 \left( \frac{x}{2} \right)^3 - 666.66 \left( \frac{x}{2} \right)^2 \right) \frac{1}{E_x} = \frac{-1333.33x^3}{E_x} \]

\[ \gamma_{\text{max}} = -1.89 \text{ inches} \]
5 Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: “Influence Line for the Vertical Reaction at A”.) Hint: The answer is not really “Influence Line for the Vertical Reaction at A”. 😊
1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an “equal” or “not equal” sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

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2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?

\[
\begin{align*}
W &= \sigma \\
\gamma(x) &= C_1 \\
M(x) &= C_1 x + C_2 \\
\varepsilon_2(x) &= C_1 x^2 + C_2 x + C_3 \\
\varepsilon_1(x) &= \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4 \\
\end{align*}
\]

10 3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the absolute maximum (positive or negative) moment at B generated by the loads shown.

\[
M_B = \frac{5(10)}{2} \left[ \frac{5 k \cdot ft}{k \cdot ft} + 1 k \cdot ft \right] \\
- \frac{10(20)}{2} \left[ 1 k \cdot ft \right] \\
- 7 \left[ 10 \right] \\
150 - 100 - 70 \\
M_B = -20 k \cdot ft
\]

\[
M_B = -20 k \cdot ft
\]
4. [35 pts] Given the following beam, draw the influence lines for:
- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

\[ \Sigma F_y = 0 = A_y + C_y - 1 \]
\[ A_y + C_y = 1 \]

\[ \Sigma M_A = 0 = C_y (3) \]
\[ C_y = 0 \]
\[ A_y = 1 \]

\[ y = -\frac{1}{30}x + 1 \]
\[ y = \frac{1}{30}(30) + 1 \]
\[ y = \frac{1}{3} \]
b) release support at C

\[ \sum M_A = 0 = C_y(30) - 1(70) \]
\[ C_y = 1 \]

Vertical reactions at C

\[ \sum F_y = 0 = A_y + C_y - 1 \]
\[ A_y + C_y = 0 \]
\[ \sum M_A = 0 = 1(10) - C_y(30) \]
\[ C_y = \frac{1}{3} \]
\[ A_y = \frac{2}{3} \]

C)

\[ M_B = 0 = M_B - \frac{2}{3}(10) \]
\[ M_B = 20/3 \]
\[ \sum M_B = 0 = M_B - \frac{1}{3}(20) \]
\[ M_B = 20/3 \]

\[ y = \frac{1}{3}(30) + \frac{20}{3} (0, \frac{20}{3}) (20, 0) \]
\[ \frac{20}{3} - 0 \]
\[ 0 - \frac{20}{3} = \frac{2}{2} \]
\[ \frac{1}{3} = -\frac{1}{3} \]
5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let $E=29,000$ ksi and $I=4200$ in$^4$. Your answer should be given in inches.

\[ E = 29,000 \text{ ksi} \]
\[ I = 4200 \text{ in}^4 \]

\[ Y = \frac{20}{x} \]
\[ I(x) = x \]
\[ \gamma = \frac{1}{10} \]

\[ \delta = \frac{10}{x^2} \]
\[ \nu(20) = \frac{1}{10} \]
\[ \nu(0) = \frac{1}{2} \]

\[ M_B = -1333.33 \]

\[ M_B = \frac{-1333.33}{x} \]

\[ M_B = -1333.33 - 100x + \frac{x^3}{12} \]

\[ \theta(x) = \frac{x^4}{48} + 50x^2 - 1333.33x + C_1 \]

\[ \theta(x = 0) = 0 = 0 + 0 - 0 + C_1 \]

\[ C_1 = 0 \]

\[ \theta(x) = \frac{x^4}{48} + 50x^2 - 1333.33x \]

\[ \theta = -\frac{x^4}{48} + 50x^2 - 1333.33x \]
\[ E I Y(x) = -\frac{x^5}{240} + \frac{50}{3} x^3 - 666.67x^2 + C_2 \]

\[ E I Y(x=0) = 0 + 0 + 0 + C_2 = C_2 \]

\[ E I Y(x) = -\frac{x^5}{240} + \frac{50}{3} x^3 - 666.67x^2 \checkmark \]

\[ E I Y(x=20) = -\frac{20^5}{240} + \frac{50}{3}(20^3) - 666.67(20^2) \checkmark \]

\[ = \frac{13,333.33 + 13,333.33}{2} = 26,666.67 \]

\[ Y_{\text{max}} = 315.27 \text{in} \]

\[ \omega = \frac{1}{2} x \]

\[ V(x) = -\frac{x^2}{4} + C_1 \]

\[ U(x=0) = 100 = 0^2 + 0 + C_1 \]

\[ V(x) = -\frac{x^2}{4} + 100 \]

\[ M(x) = -\frac{x^3}{12} + C_1 x + C_2 \]

\[ M(x=0) = -1333.33 \]

\[ M(x) = -\frac{x^3}{12} + 100x - 1333.33 \]

\[ \text{Something wrong in conversion of math...} \]
Bonus A [5 pts] — Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: “Influence Line for the Vertical Reaction at A”). Hint: The answer is not really “Influence Line for the Vertical Reaction at A”. ☺

[Diagram of a beam with labels A, B, C, D, showing an internal hinge and influence lines for the moment at A.]
1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

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2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?

\[ \frac{A - \frac{E}{3} v}{\beta} \]
\[ v(x) = C_1 \]
\[ M_0(x) = C_2 \]
\[ \delta = C_3 \]
\[ \gamma(x) = C_4 \]

4 constants

10. [10 pts] Given the following influence line for moment at B on the following beam, calculate the moment at B generated by the loads shown.

\[ M_B = (1 \times \delta)(5 \times 5\delta) + (5 \times \delta)(5 \times 5\delta) - (1 \times \delta)(10 \times 10\delta) - (7 \times 10) \]

\[ M_B = -20 k \cdot ft \]
4. [35 pts] Given the following beam, draw the influence lines for:
- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

\[ \sum F_A = 0 = (1k)(x) + C_V(30') \]
\[ \therefore C_V = \frac{x}{30} \]

\[ \sum M_C = 0 = (1k)(30-x) - A_Y(30) \]
\[ \therefore A_Y = 1 - \frac{x}{30} \]

1. L. for reaction at A

1. L. for reaction at C
\[ G + 2 M_B = 0 = -M_B + C_T \text{ (20m)} \]
\[ M_B = 20 \, C_T \]
\[ M_B = \frac{2x}{3} \quad 0 \leq x \leq 10' \]

\[ \sum M_B (10') = 0 = M_B - (A_T)(10') \]
\[ M_B = 10 \, A_T \]
\[ M_B = 10 - \frac{x}{3} \quad 10' \leq x \leq 40' \]

For moment at B
\[ M_B = 20 \phi \]
\[ M_B = 10 - \frac{x}{3} \]
\[ -10 \frac{1}{2} \]
5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let \( E = 29,000 \) ksi and \( I = 4200 \) in\(^4\). Your answer should be given in inches.

\[
\begin{align*}
\omega(x) &= (10 - \frac{3}{2}x) \quad \checkmark \\
V(x) &= \int -\omega(x) \, dx \\
&= \int -\left(10 - \frac{3}{2}x\right) \, dx \\
&= \int -10 + \frac{3}{2}x \, dx \\
V(x) &= \frac{x^2}{4} - 10x + C_1 \\
M(x) &= \int V(x) \, dx \\
&= \int \left(\frac{x^2}{12} - \frac{5x^2}{2} + C_1 x + C_2\right) \, dx \\
M(x) &= \frac{x^3}{12} - \frac{5x^3}{6} + C_1 x^2 + C_2 x \\
\text{EI} \ \Theta(x) &= \int \frac{M(x)}{EI} \, dx \\
&= \int \left(\frac{x^3}{12} - \frac{5x^3}{6} + C_1 x^2 + C_2 x\right) \, dx \\
\Theta(x) &= \frac{x^4}{48EI} - \frac{5x^4}{24EI} + \frac{C_1 x^3}{3} + C_2 x + C_3 \\
\gamma(x) &= \int \Theta(x) \, dx \\
&= \frac{1}{EI} \int \left(\frac{x^4}{48EI} - \frac{5x^4}{24EI} + \frac{C_1 x^3}{3} + C_2 x + C_3\right) \, dx \\
\gamma(x) &= \frac{1}{EI} \left(\frac{x^5}{240} - \frac{5x^4}{12} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4\right) \\
\text{B.C.} &\quad \gamma(x = 0) = 0 \\
&\quad C_1 = \frac{C_2^2}{4} - 10(0) + C_1 \quad \therefore C_1 = 0 \\
&\quad M(x = 0) = 0 \\
&\quad 0 = C_3^2 - 3(0)^2 + C_2(0) + C_2 \quad \therefore C_2 = 0 \\
&\quad \Theta(x = 20) = 0 \\
&\quad 0 = \frac{1}{240} \left(\frac{20^5}{48} - \frac{5(20)^4}{24} + C_2\right) \\
&\quad 0 = \frac{1600000}{48} - \frac{400000}{3} + C_3 \\
&\quad 0 = \frac{1600000}{48} - \frac{640000}{12} + C_3 \\
&\quad \therefore C_3 = 100000 \\
&\quad \gamma(x = 20) = 0 \\
&\quad 0 = \frac{1}{240} \left(\frac{20^5}{48} - \frac{5(20)^4}{24} + 10000(20) + C_4\right) \\
&\quad 0 = \frac{3200000}{240} - \frac{800000}{12} + 200000 + C_4 \\
&\quad 0 = \frac{3200000}{240} - \frac{1600000}{240} + \frac{1600000}{240} + C_4 \\
&\quad \therefore C_4 = -\frac{3520000}{240} \\
\end{align*}
\]
\[ y(x) = \frac{1}{EI} \left( \frac{x^5}{240} - \frac{5x^4}{12} + 10000x - \frac{352000000}{240} \right) \]

\[ y_{\text{max}}(x=0) = \frac{1}{EI} \left( \frac{0^5}{240} - \frac{5(0)^4}{12} + 10000(0) - \frac{352000000}{240} \right) \]

\[ E = \frac{29,000}{\text{in}} \times \frac{144}{1} \frac{\text{in}^2}{\text{ft}^2} = 4176000 \text{ k/ft} \]

\[ I = 4200 \text{ in}^4 \times \left( \frac{144}{12 \text{ in}} \right)^4 \]

\[ I = \frac{4200}{20738} \]

\[ y_{\text{max}} = -\frac{352000000}{2400 \left( \frac{4176000 \text{ k/ft}}{20738} \right) \left( \frac{144}{12 \text{ in}} \right)^4} \]

\[ y_{\text{max}} = -0.173 \text{ ft} \approx -2.08 \text{ in} \]
Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: "Influence Line for the Vertical Reaction at A"). Hint: The answer is not really "Influence Line for the Vertical Reaction at A".

![Diagram of a beam with an internal hinge at B and an influence line for the moment at A marked]
1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an “equal” or “not equal” sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

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<td>$\theta \neq 0$</td>
<td>$V \neq 0$</td>
<td>$\frac{simpla}{3}$</td>
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2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?

\[
V = \int \omega \, dx \\
M = \int V \, dx \\
\theta = \frac{1}{E I} \int M \, dx \\
N = \int \sigma \, dx
\]

4 integrations per section \times 
4 sections to analyze

16 integration constants

3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the moment at B generated by the loads shown.

\[M_B = \left( \frac{2}{3} \cdot 10.5 \right) - \left( 7 \cdot 10 \right) = \left( \frac{20}{3} - 70 \right) \text{ kip-ft} \]

\[M_B = -20 \text{ ft-kip} \]
4. [35 pts] Given the following beam, draw the influence lines for:
   - (10/35) Vertical reaction at A
   - (10/35) Vertical reaction at C, and
   - (15/35) Moment at point B.
   You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

   ![Beam Diagram]

   Use equilibrium method
   Overall reaction:
   \[ \Sigma M_A = 0 \]
   \[ 30 \cdot C_y - x = 0 \]
   \[ C_y = \frac{x}{3} \]
   \[ \Sigma F_y = 0, \quad A_y + C_y - 1 = 0 \]
   \[ A_y = 1 - \frac{x}{30} \]

   Break beam, solve for moment at B
   \[ \Sigma M = 0 \]
   \[ -(1 - \frac{x}{30}) \cdot 10 + 1 \cdot (10 - x) + M_B = 0 \]
   \[ M_B = 10 - \frac{x}{3} - 10 - x \]
   \[ M_B = \frac{2x}{3}, \quad 0 < x < 10 \quad M(10^2) = \frac{2 \times 100}{3} = 66.67 \quad \checkmark \]

   \[ 0 < x < 10 \]
   \[ \Sigma M_B = 0 \]
   \[ -(1 - \frac{x}{30}) \cdot 10 + M_B = 0 \]
   \[ M_B = 10 - \frac{10x}{30} = 10 - \frac{x}{3} \]
   \[ M(10^2) = \frac{2 \times 100}{6} = 66.67 \quad \checkmark \]

   I.I. on next page.
5. [35] Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let $E=29,000$ ksi and $I=4200$ in$^4$. Your answer should be given in inches.

Let $x=0$ at $B$, positive to the left.

**Overall rxn:**

$\sum F_x = 0$

$-10 \cdot 20 + 0 = 0$

$B_x = 100$ kip

$\sum M = 0$

$(10 \cdot 10 \cdot 20) \cdot \frac{x}{2} (20) + M_B = 0$

$M_B = -1333.333$ ft-kip

$c(x) = 4a(x)$, in the defined coordinate system

$V(x) = \int c(x) dx$

$V(x) = -\frac{10}{2} x^2 + C_1$

$V(0) = 0$

$C_1 = +100$

$V(x) = -\frac{10}{2} x^2 + 100$

$M(x) = \int V(x) dx$

$M(x) = -\frac{10}{2} x^3 + 100x + C_2$

$M(0) = +1333.333$ kip

$C_2 = +1333.333$

Check: $M(20) = -\frac{10}{2} \cdot 20^3 + 100 \cdot 20 \mp 1333.333 = 0$

$\Theta = \frac{1}{E} \int M(x) dx$

$\Theta(x) = \frac{1}{E} \left( -\frac{10}{2} x^4 + 50x^2 + 1333x + C_3 \right)$

$\Theta(0) = 0$, $C_3 = 0$

$\overrightarrow{C(x)} = \int \overrightarrow{\Theta(x)} dx$

$\overrightarrow{C(x)} = \frac{1}{E} \left( \frac{10}{2} x^5 + \frac{50}{2} x^3 + 1333x + C_4 \right)$

$\overrightarrow{C(0)} = 0$, $C_4 = 0$
\[ N(x) = \frac{-1}{EI} \left( \frac{1}{240} x^5 - \frac{50}{3} x^3 + \frac{1000}{3} x \right) \]

\[ EI = 29000 \text{ ksi} \cdot 4200 \text{ in}^4 = 1218.16 \text{ ksi} \cdot \text{in}^4 \cdot \frac{1 \text{ ksi} \cdot \text{in}^4}{\text{in}^4} = 8.456 \times 10^6 \text{ kip ft}^2 \]

May deflection at end \((x=20')\)

\[ N(20) = \frac{-1}{EI} \left( \frac{1}{240} \cdot 20^5 - \frac{50}{3} \cdot 20^3 + \frac{1000}{3} \cdot 20 \right) \]

\[ N(20) = -0.1734 \text{ ft} = -2.0 \text{ in} \quad \checkmark \]
5. Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: “Influence Line for the Vertical Reaction at A”.) Hint: The answer is not really “Influence Line for the Vertical Reaction at A”.

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Diagram:

- Beam labeled A, B, C, D with an internal hinge.
- Influence line diagram showing a horizontal line for a moment at A.
1. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an “equal” or “not equal” sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

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2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?

From A to B: 4 const
From B to C: 4 const
From C to D: 4 const
From D to E: 4 const

\[ \therefore 16 \text{ const of integration} \]

3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the moment at B generated by the loads shown.

\[ M_B = \left( \frac{5}{3} \right) \left( \frac{1}{2} \right) \left( \frac{5\, \text{kips}}{5\, \text{ips}} \right) \]

\[ + \left( \frac{1}{2} \right) \left( \frac{1}{3} \right) \left( -10 \right) \left( 10 \right) \]

\[ M_B = 1250 + 250 - 2000 - 70 \]

\[ M_B = -570\, \text{kips} \times \]

\[ -7 \]
4. [35 pts] Given the following beam, draw the influence lines for:
   - (10/35) Vertical reaction at A
   - (10/35) Vertical reaction at C, and
   - (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

Release the reaction @ A.

Release the reaction @ C.

Release the reaction @ B.

---
5. Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let $E=29,000$ ksi and $I=4200$ in$^4$. Your answer should be given in inches.

$$V(0) = 0$$
$$M(0) = 0$$
$$\theta(0) = 0$$
$$y(0) = 0$$

$$u(x) = \frac{1}{2}(10)(20-x)$$
$$w(x) = 100 + 5x$$
$$V(x) = \int w(x) \, dx$$
$$V(0) = 100 + 5x$$
$$V(x) = -100x + \frac{5}{2}x^2 + C_1$$
$$B.C. 1 \ V(0) = 0$$
$$-100(0) + \frac{5}{2}(0)^2 + C_1 = 0$$
$$C_1 = 0$$

$$V(2) = -100x + \frac{5}{2}x^2$$

$$M(x) = \int V(x) \, dx$$
$$M(0) = \int (-100x + \frac{5}{2}x^2) \, dx$$
$$M(x) = -100x^2 + \frac{5}{6}x^3 + C_2$$
$$B.C. 2 \ M(0) = 0$$
$$-100(0)^2 + \frac{5}{6}(0)^3 + C_2 = 0$$
$$C_2 = 0$$

$$M(x) = -100x^2 + \frac{5}{6}x^3$$

$$EI \ \theta(y) = \int M(x) \, dx$$
$$\theta(y) = \frac{1}{EI} \int (50x^2 + \frac{5}{6}x^3) \, dx$$
$$\theta(0) = \frac{1}{EI} \left( \frac{50}{3}x^3 + \frac{5}{24}x^4 + C_3 \right)$$
$$B.C. 3 \ \theta(2) = 0$$
$$\frac{1}{EI} \left( \frac{50}{3}(2)^3 + \frac{5}{24}(2)^4 + C_3 \right) = 0$$
$$C_3 = -100,000$$

$$\therefore \ \theta(0) = \frac{1}{EI} \left( \frac{50}{3}x^3 + \frac{5}{24}x^4 + 100,000 \right)$$

$$y(x) = \frac{1}{EI} \int \theta(x) \, dx$$
$$y(0) = \frac{1}{EI} \left( \frac{50}{3}x^3 + \frac{5}{24}x^4 + 100,000 \right)$$

$$y(2) = \frac{1}{EI} \left( \frac{50}{12}x^4 + \frac{5}{120}x^5 + 100,000x + C_4 \right)$$
$$B.C. 4 \ y(2) = 0$$
$$\frac{1}{EI} \left( \frac{50}{12}x^4 + \frac{5}{120}x^5 + 100,000x + C_4 \right) + 100,000(2) + C_4 = 0$$
$$C_4 = -1,466,666$$

$$\therefore \ y(x) = \frac{1}{EI} \left( \frac{50}{12}x^4 - \frac{5}{120}x^5 - 100,000x - 1,466,666 \right)$$

Next page
\[ y_{\text{max}} @ \theta = 0, \text{ or in this case at the free end} \]
\[ y_{\text{max}} @ x = 0 \]

\[ y_{\text{max}} = \frac{1}{E_{t}} \left( \frac{25}{120} y + \frac{5}{120} x^2 \right) + 100,000(0) - 1,466,666.7 \]
\[ y_{\text{max}} = \frac{-1466666.7(12)}{29000(4200)} \]
\[ y_{\text{max}} = -20.8'' \text{ to big}! \]
No sure what I did wrong?

\[ w = 10 - \frac{1}{2} x \]
\[ UX = \frac{-1}{2} x x \]
\[ V(x) = \frac{-1}{2} x d x \]
\[ V(x) = \frac{-1}{4} x^2 + C_1 \]
\[ C_1 = 0 \checkmark \]
\[ V(x) = \frac{-1}{4} x^2 \]
\[ M(x) = \frac{-1}{4} x^2 d x \]
\[ M(x) = \frac{-1}{6} x^3 + C_2 \]
\[ C_2 = 0 \checkmark \]
\[ M(x) = \frac{-1}{6} (x^3) \]

\[ EIG = \frac{1}{6} x^3 d x \]
\[ \theta_{(x)} = \frac{1}{E_{t}} \left( \frac{-25}{120} x^4 + C_3 \right) \]
\[ \theta_{(x)} = 0 \checkmark \quad 0 = \frac{-25}{120} (20^3) + C_3 \quad C_3 = 6667 \]
\[ \theta_{(x)} = \frac{1}{E_{t}} \left( \frac{5}{24} x^4 + 6667 \right) \]

\[ y(x) = \frac{1}{120} x^5 - 6667 d x \]
\[ y_{(x)} = \frac{1}{120} x^5 + 6667 + C_4 \]
\[ y_{(0)} = 0 = \frac{1}{120} (20^5) + 6667(2.5) + C_4 \]
\[ C_4 = -106673 \]
\[ y(x) = \frac{1}{120} \left( -\frac{5}{24} x^5 + 6667(x) + 106673 \right) \]
\[ y_{(0)} = \frac{106673(2.5)}{29000(4200)} = 1.51'' \]
\[ \boxed{y_{\text{max}} = 1.51''} \]
Load function incorrect
Bonus A [5 pts] – Given the following beam, what value does the influence line describe, and where on the beam does it describe it? (A sample answer would be in the format: “Influence Line for the Vertical Reaction at A”.) Hint: The answer is not really “Influence Line for the Vertical Reaction at A”. 😋
14. [15 pts] Fill in the right three columns of the following table. For the middle two columns, place an "equal" or "not equal" sign in the blanks. Then draw the type of conjugate support corresponding to the real support in the right-most column.

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2. [5 pts] If you were using the method of direct integration to find the equations of the elastic curve for the entire length of the beam shown below (starting from the load function, not the moment function), how many constants of integration would you have to account for as part of your solution?

We would have to account for 4 constants of integration in the solution.

3. [10 pts] Given the following influence line for moment at B on the following beam, calculate the moment at B generated by the loads shown.

\[ M_{max} = \frac{1}{2} (5)(10)(1) + \frac{1}{2} (5)(10)(5) - \frac{1}{2} (10)(20)(1) - 7(10) \]

\[ M_{max} = 25 + 125 - 100 - 70 \]

\[ M_{max} = -20 \text{ kip-ft} \]

\[ \text{good!} \]
4. [35 pts] Given the following beam, draw the influence lines for:
- (10/35) Vertical reaction at A
- (10/35) Vertical reaction at C, and
- (15/35) Moment at point B.

You may use either the equilibrium method or Mueller-Breslau's Principle to complete the problem, but please clearly state which you have chosen.

M.B. method: small deflection of released structure at A.
apply 1 k load to A.

using similar triangles to find ordinates at B & D:
\[ \frac{1}{30} = \frac{B}{20} \quad B = \frac{y_B}{3} \]
\[ y_B = 3 + x \]
\[ y_0 = 2 \]
\[ y_{(10)} = \frac{20}{10} x + 0 = y_B \]

M.B. method: small deflection of released structure at C. apply 1 k load at C to find ordinate at C.

Using similar triangles to find other ordinates:
\[ \frac{1}{30} = \frac{C}{10} \quad C = \frac{y_C}{3} \]
\[ y_C = \frac{1}{2} x + 0 \]
\[ y_3 = \frac{1}{20} x \]

M.B. - insert an internal pin at B & apply small rotation keeping relative coordinates rigid.
Find ordinates at E.L.

\[ M_B = 10A_1 \quad 10' \leq x \leq 40' \]

\[ M_B = \begin{cases} 
C_y(20) = \frac{x}{20} (20) = \frac{x}{2} & 0' \leq x \leq 10' \\
A_y(10) = \left( \frac{x}{50} + 1 \right)(10) = -\frac{1}{5} x + 10 & 10' \leq x \leq 40' 
\end{cases} \]

\[ \bar{y}(x) = \frac{20/5}{1} \]

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CRB
5. (35) Determine the maximum deflection of the following loaded beam under the given loads, using the method of direct integration. You may start with the load function or the moment function. Let $E=29,000$ ksi and $I=4200$ in$^4$. Your answer should be given in inches.

\[ +1.5 = 31.5 \text{ (See note on last page)} \]

\[ W = \frac{1}{2}x \left( \frac{1}{3}x^3 \right) = \frac{1}{6}x^2 \]

\[ V = \frac{1}{EI} \left( 1335.33 \frac{x^2}{2} - C_1 \right) \]

\[ V = \frac{1}{EI} \left( \frac{1335.33}{2} x^2 \right) \]

\[ \text{Maximum vertical deflection will be at the end of the beam (free end)} \]

\[ \delta = \frac{1}{EI} \left( \frac{1335.33}{2} \right) \left( 20 \text{ ft} \right)^2 = \frac{667665 \text{ in}^3}{29000 \text{ ksi} \times 4200 \text{ in}^4} = 0.24 \text{ in.} \]
But the load is a straight line, not a parabola

\[ w = \frac{1}{2} x \] 

\[ V = \int -w \, dx = \int - \frac{1}{2} x^2 \, dx = -\frac{1}{3} x^3 + C_1 \]

You included here...?

\[ \frac{w}{100} = -\frac{1}{2} x \]

B.C. \( V(x=0) = 100 \)

\[ V(x) + 100 = -\frac{1}{2} x^2 + C_1 \]

\[ R.C. \quad V(x=10) = -1333.33 \]

\[ -1333.33 = -\frac{1}{2} \times (10)^2 + 100(10) = C_1 \]

\[ C_1 = 100 \]

\[ \frac{w}{100} = -\frac{1}{2} x \]

B.C. \( V(x=0) = 100 \)

\[ V(x) = -\frac{1}{2} x^2 + 100x + c_2 \]

\[ R.C. \quad V(x=10) = -1333.33 \]

\[ -1333.33 = -\frac{1}{2} (10)^2 + 100(10) = c_2 \]

\[ c_2 = -1333.33 \]

\[ \theta = \frac{1}{2E} \left( \frac{-1333.33}{2} \right) \]

\[ \theta = \frac{1}{2E} \left( \frac{-1333.33}{2} \right) \]

\[ \gamma = \frac{1}{E} \left( \frac{-1333.33}{2} \right) \]

\[ \gamma = \frac{1}{E} \left( \frac{-1333.33}{2} \right) \]

Max deflection occurs at the free end of the beam (x=10)

\[ V(x=10) = \frac{1}{E} \left( -17777.71 \right) = \frac{-17777.71 \text{in}}{29000 \text{in}^3} = -0.710 \text{in} \]

\[ V(x=10) = -17777.71 \text{in} \]

\[ V(x=10) = -17777.71 \text{in} \]

\[ \gamma(x=10) = -2.52 \text{in} \]

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[Diagram of a beam with an internal hinge and a graph of influence lines labeled as M_a.]
\[ \psi(x) = -Wx + c_1 \]

\[ W = \int \psi(x) \, dx = -\frac{Wx^2}{2} + 100x + c_2 \]

\[ E = \int \psi(x) \, dx = -\frac{Wx^3}{6} + 100x^2 - 1333.33x + c_3 \]

\[ \frac{dE}{dx} = \left( -\frac{Wx^2}{6} + 50x^2 - 1333.33x \right) = 0 \]

\[ c_3 = 1333.33 \]

\[ E = \int \psi(x) \, dx = \frac{1}{2} \left( -\frac{Wx^4}{24} + \frac{50x^3}{3} - 1333.33x^2 + c_4 \right) \]

\[ \psi(x) = \frac{1}{2} \left( -\frac{Wx^4}{24} + \frac{50x^3}{3} - 1333.33x^2 \right) \]

If \( W = \frac{1}{2} \), then

\[ \psi(x) = \frac{1}{2} \left( -\frac{W^5}{48} + \frac{50W^3}{2} - 1333.33W^2 \right) \]

\[ \psi(x = \theta) = \frac{1}{2} \left( -\frac{10^5}{48} + \frac{50(10)^3}{2} - 1333.33(10)^2 \right) \]

\[ = \frac{-199,149.32}{2} = \frac{-199,999.33}{20,000} \times \frac{12}{12} = -228.86 \]

This page has the correct solution, however, I cannot give you full credit because of the dual solutions. You correctly identified the problem, though, 4/5 I'll reduce the points deducted to 2/5 of that. +1.5