76/100

CE 461 – Structural Analysis
EXAM No. 3

Name: ______________________

4

1. [6 pts] Identify three advantages to using indeterminate structures over determinate structures.

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added stublity b/c of the redunda-

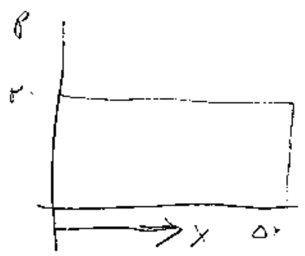
Not so much added stability as Redundancy

Less shear strain in an Ind. structure w/ the same loading ? Not necessarily...

Less deformation in an Ind. stucture w/ the same loading ✓

2

2. [9 pts] Explain the presence of the ½ factor in the equation $U_e = \frac{1}{2}P\Delta$, which describes the amount of external work performed by a load which is applied to a structure gradually.



as you apply a load gradually the previous load amount "goes along for the ride".

$U = \int_0^{\Delta} P \, d\Delta$ – This is correct but, what does this means? (How does the ½ factor come into play?)

15

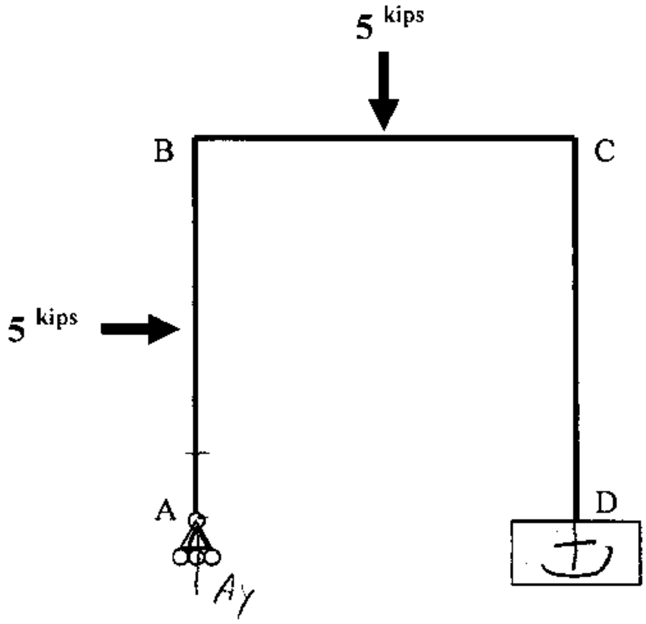
3. [15 pts total] Given the following indeterminate frame to be analyzed using the force/flexibility method, with a final goal of solving for all support reactions:

2 a. [2 / 15 pts] Identify the degree of indeterminacy,

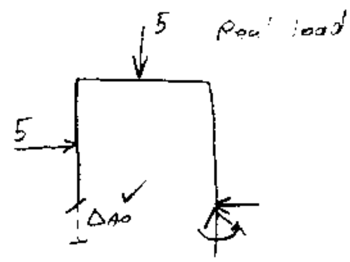
3 b. [3 / 15 pts] Identify the redundant(s) you would choose to solve for so that you could complete the analysis, and

10 c. [10 / 15 pts] Write the compatibility equation(s) to solve for those redundants.

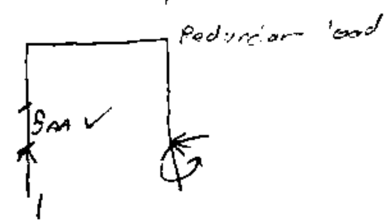
(Please note that you do not actually have to solve for the redundants to complete this problem.)



a) EQS 3 + 0ec < UNK 4 ∴ 1° IND ✓



b) Remove A_y OK

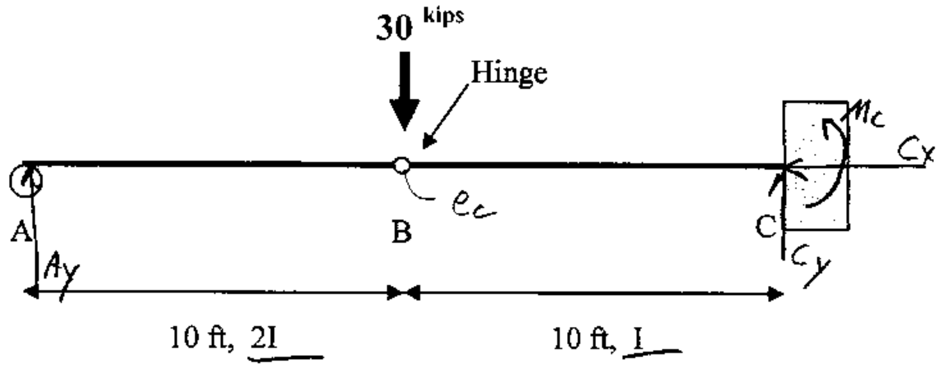


c) $\Delta A = 0 = \Delta_{A0} + \delta_{AA} A_y$ ✓

$A_y = -\frac{\Delta_{A0}}{\delta_{AA}}$ GOOD

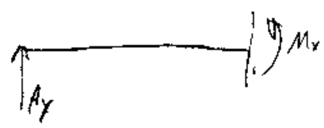
27

4. [30 pts] Solve for the vertical deflection at point B on the following beam. Use the conjugate beam method. Let $E = 4{,}000$ ksi and $I = 3{,}000$ in^4.



$\Sigma M_c = 0 = M_c + 30(10)$

$M_c = -300$

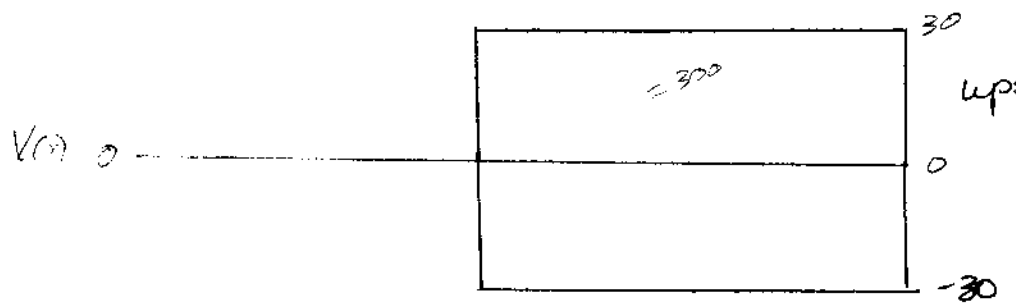


$\Sigma F_y = 0 = A_y$

$A_y = 0$

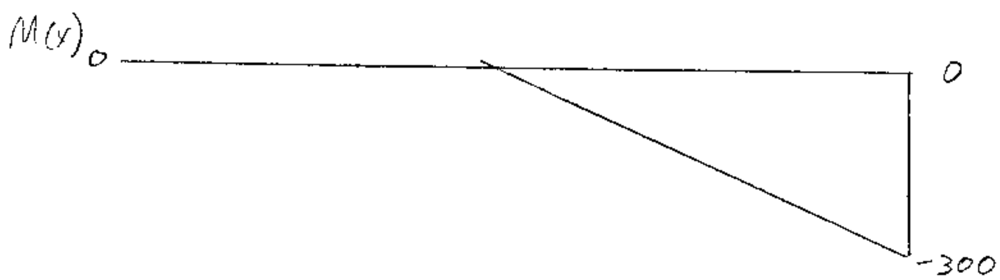
$C_y = 30$

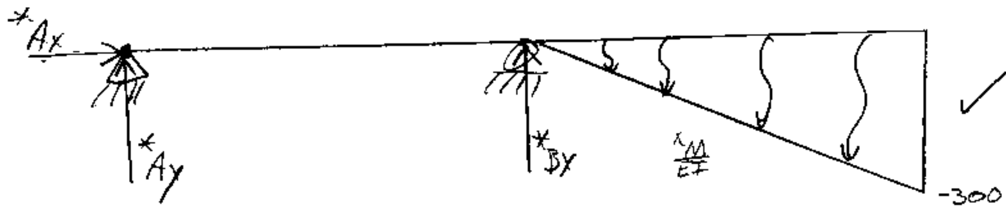
$C_x = 0$

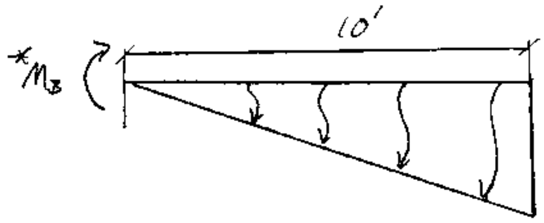


upside-down

-2







$$\Sigma M_x = 0 = {}^*M_B + \left(300(10)\tfrac{1}{2}\right)\left(\frac{10(2)}{3}\right)$$

$${}^*M_B = \frac{10{,}000}{EI}$$

$$\Delta_B = {}^*M_B = \frac{10{,}000}{EI}$$

$$\Delta_B = \frac{10000(12^3)}{4000(3000)}$$ ✓

$$\frac{(k\text{-}ft)(12^2/ft)}{(k/in^2)\, in^4}$$

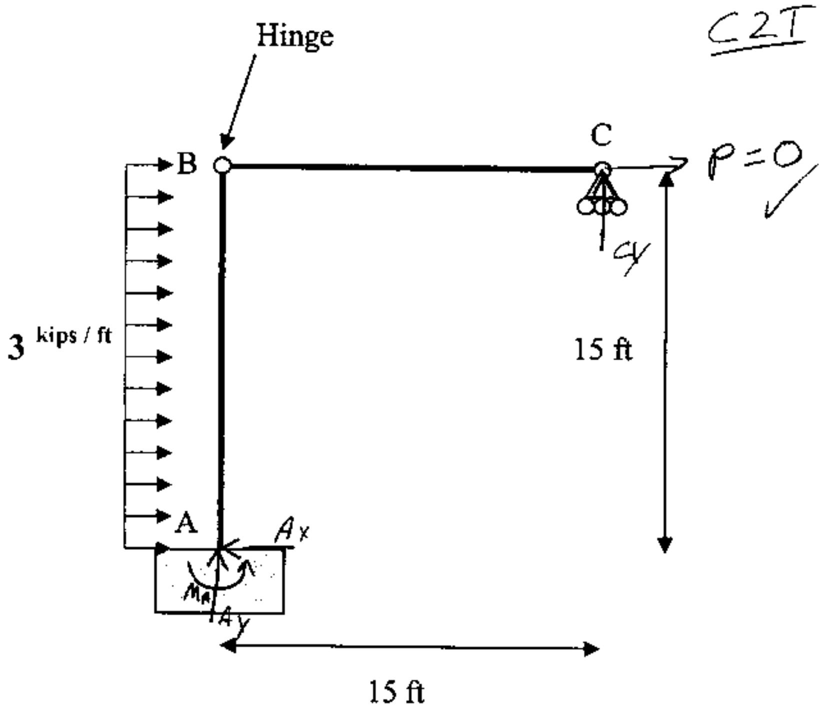
$$\boxed{\Delta_B = 1.44\ in}$$

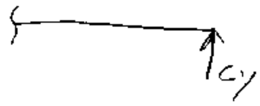
which direction? (-1)

26

5. [40 pts] Solve for the horizontal deflection at point C using either the virtual work method or Castigliano's second theorem. Be sure to clearly note which method of analysis you have chosen. Include shear and moment diagrams for the real force system shown below as part of your solution. Let E = 29,000 ksi and I = 2,000 in^4. EI is constant for the entire frame.

C2T





$\Sigma F_y = 0 = C_y$

$C_y = 0$

Rxns

$\Sigma F_x = 0 = -A_x + 3(15) + P$

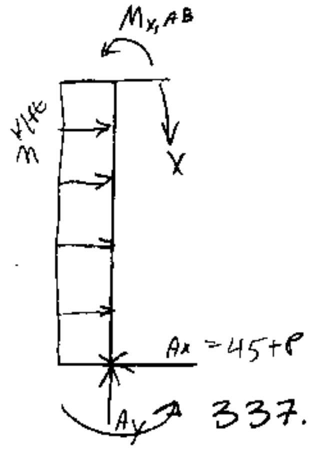
$A_x = 45 + P$ ✓

$\Sigma M_A = 0 = M_A - 3(15)\left(\frac{15}{2}\right) - P(15)$

$M_A = 337.5 + 15P$ ✓

$\Sigma F_y = 0 = A_y + C_y$

$C_y = 0 \therefore A_y = 0$ ✓



$$\sum M_X = 0 = M_{X,AB} + 3(X)\left(\frac{X}{2}\right) - (45+P)(X)$$

$$M_{X,AB} = \frac{3}{2}X^2 - 45X + PX$$

337.5 + 15P Incomplete FBD −7

$$\sum M_X = 0 = M_{X,BC}$$

$$M_{X,BC} = 0$$

Seg	Org	Limit	M(x)	$\frac{\partial M}{\partial P}$
AB	A	0–15	$\frac{3}{2}x^2 - 45x - Px$ x	$-x$ x
BC	B	0–15	0	0

$$\Delta_C = \frac{1}{EI}\int_0^{15}(-x)\left(\frac{3}{2}x^2 - 45x - \cancel{Px}\right)dx$$

$$\Delta_C = \frac{1}{EI}\int_0^{15} -\frac{3}{2}x^3 + 45x^2 \, dx$$

$$\Delta_C = \frac{1}{EI}\left(-\frac{3}{8}x^4 + 15x^3 \Big|_0^{15}\right)$$

$$\Delta_C = \frac{1}{EI}\left(-18984.4 + 50625\right)$$

$$\Delta_C = \frac{31640.6}{EI}$$

$$\Delta_C = \frac{(31640.6)(12^3)}{(29000)(2000)}$$

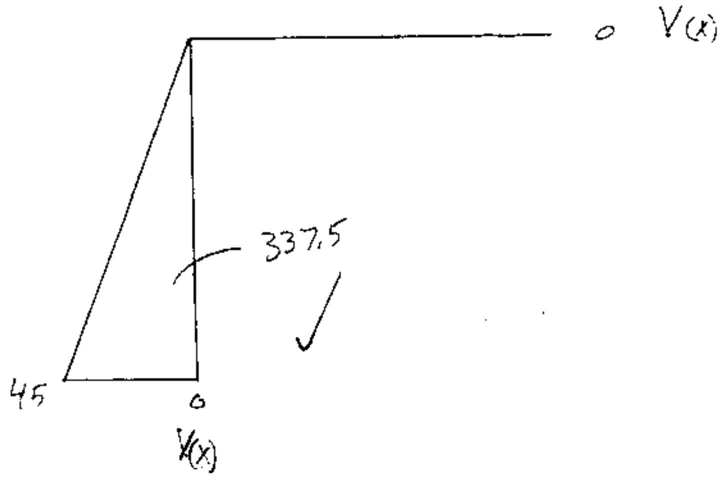
$$\boxed{\Delta_C = 0.94\,in}$$

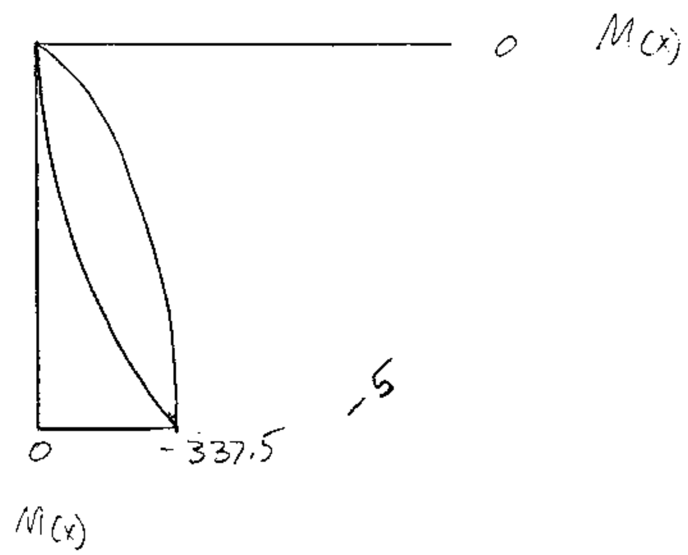
$M_A = 337.5$

$A_x = 45 \leftarrow$

$A_y = 0$

$C_y = 0$





0

Bonus (5 pts): Derive the following equation, which describes the amount of internal strain energy that develops in an axially loaded member when the member is gradually loaded with a force, in three steps or less.

$$\boxed{U_i = \frac{N^2 L}{2AE}}$$

$U_e = U_i$

Internal Work (Strain Energy) for Axial Forces $U_i = \dfrac{N^2 L}{2AE}$

Internal Work (Strain Energy) for Bending $U_i = \displaystyle\int_0^L \frac{M^2 dx}{2EI}$

Method of Virtual Work Applied to Beams
$$\begin{cases} 1(\Delta) = \left[\displaystyle\int_0^L \frac{m_v M}{EI} dx\right] + \left[\displaystyle\int_0^L \frac{v_v V}{GA_v} dx\right] \\ 1(\theta) = \left[\displaystyle\int_0^L \frac{m_v M}{EI} dx\right] + \left[\displaystyle\int_0^L \frac{v_v V}{GA_v} dx\right] \end{cases}$$

Method of Virtual Work Applied to Frames
$$\begin{cases} 1(\Delta) = \left[\sum n_v \left(\dfrac{NL}{AE}\right)\right] + \left[\sum \displaystyle\int_0^L \frac{m_v M}{EI} dx\right] + \left[\sum \displaystyle\int_0^L \frac{v_v V}{GA_v} dx\right] \\ 1(\theta) = \left[\sum n_v \left(\dfrac{NL}{AE}\right)\right] + \left[\sum \displaystyle\int_0^L \frac{m_v M}{EI} dx\right] + \left[\sum \displaystyle\int_0^L \frac{v_v V}{GA_v} dx\right] \end{cases}$$

Castigliano's Second Theorem Applied to Trusses $\Delta = \sum \left(\dfrac{\partial N}{\partial P}\right) \dfrac{NL}{AE}$

Castigliano's Second Theorem Applied to Beams
$$\begin{cases} \Delta = \left[\displaystyle\int_0^L \left(\frac{\partial M}{\partial P}\right) \frac{M}{EI} dx\right] + \left[\displaystyle\int_0^L \left(\frac{\partial V}{\partial P}\right) \frac{V}{GA_v} dx\right] \\ \theta = \left[\displaystyle\int_0^L \left(\frac{\partial M}{\partial \overline{M}}\right) \frac{M}{EI} dx\right] + \left[\displaystyle\int_0^L \left(\frac{\partial V}{\partial \overline{M}}\right) \frac{V}{GA_v} dx\right] \end{cases}$$

Castigliano's Second Theorem Applied to Frames
$$\begin{cases} \Delta = \left[\sum \left(\dfrac{\partial N}{\partial P}\right) \dfrac{NL}{AE}\right] + \left[\sum \displaystyle\int_0^L \left(\frac{\partial M}{\partial P}\right) \frac{M}{EI} dx\right] + \left[\displaystyle\int_0^L \left(\frac{\partial V}{\partial P}\right) \frac{V}{GA_v} dx\right] \\ \theta = \left[\sum \left(\dfrac{\partial N}{\partial \overline{M}}\right) \dfrac{NL}{AE}\right] + \left[\sum \displaystyle\int_0^L \left(\frac{\partial M}{\partial \overline{M}}\right) \frac{M}{EI} dx\right] + \left[\displaystyle\int_0^L \left(\frac{\partial V}{\partial \overline{M}}\right) \frac{V}{GA_v} dx\right] \end{cases}$$

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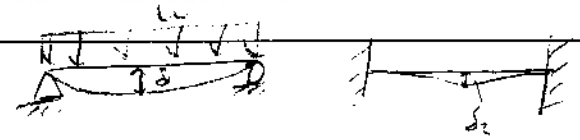
CE 461 – Structural Analysis Name: ____________________

EXAM No. 3

(6) 1. [6 pts] Identify three advantages to using indeterminate structures over determinate structures.

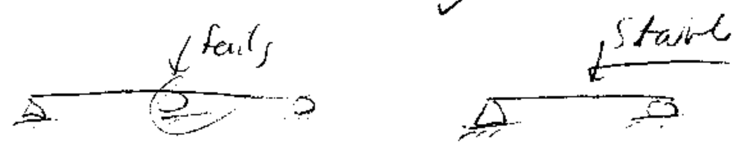
- Smaller deflections ✓ ex



$\delta_1 > \delta_2$

- If one of the supports fails it can still be stable ✓

ex



- It can support more loads w/ smaller moments ✓

(- moment Diagrams)

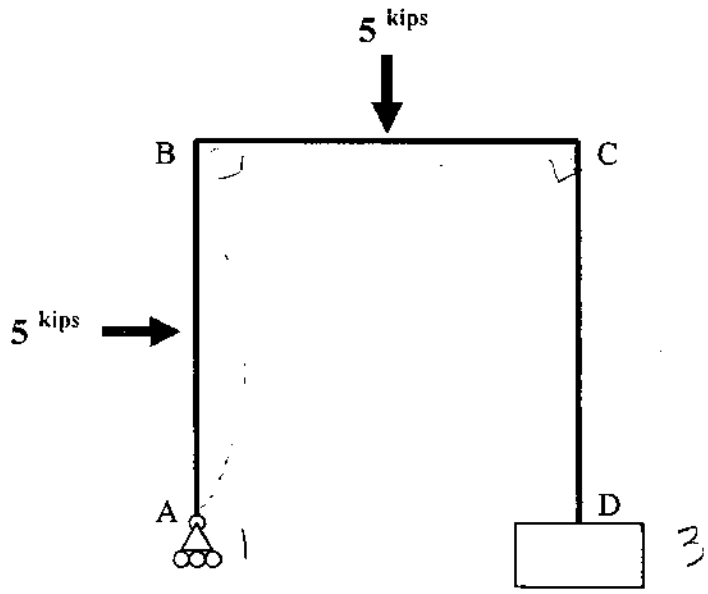
(2) 2. [9 pts] Explain the presence of the ½ factor in the equation $U_e = \frac{1}{2}P\Delta$, which describes the amount of external work performed by a load which is applied to a structure gradually.

the one half factor shows up in the equation for the amount of external work only when the load is applied gradually because it doesn't require as much work to apply a load gradually as "so, for example" beam begins deflecting it becomes easier to deflect it further or it requires less work to deflect it further. If the load is applied all at once it will require two times work to displace the bar the same amount. But why ½?

(12) 3. [15 pts total] Given the following indeterminate frame to be analyzed using the force/flexibility method, with a final goal of solving for all support reactions:

2 a. [2 / 15 pts] Identify the degree of indeterminacy,
3 b. [3 / 15 pts] Identify the redundant(s) you would choose to solve for so that you could complete the analysis, and
7 c. [10 / 15 pts] Write the compatibility equation(s) to solve for those redundants.

(Please note that you do not actually have to solve for the redundants to complete this problem.)



a) 1° Indeterminant ✓

b) you could solve for M_D, or A_y
If A_y Removed ✓

c) $\Delta_A = 0 = \Delta_{A0} + f_{AA}(A_y)$ -3

Where f_{AA} + Δ_{A0} are vertical deflections on a primary beam w/o a roller at A.
Δ_{A0} - vertical deflection from real loads f_{AA} - deflection from a unit load at A

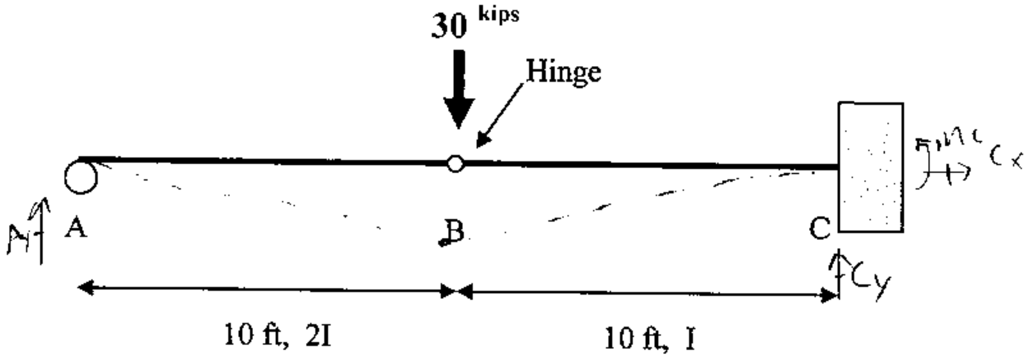
C. $\theta_D = 0 = \theta_{DO} + \alpha_{DD}(M_D)$

where θ_{DO} is the slope of a primary beam w/ a pin connection instead of a fixed connection at D

and α_{DD} is the slope of the primary beam with a unit moment applied at D

30

4. [30 pts] Solve for the vertical deflection at point B on the following beam. Use the conjugate beam method. Let E = 4,000 ksi and I = 3,000 in^4.

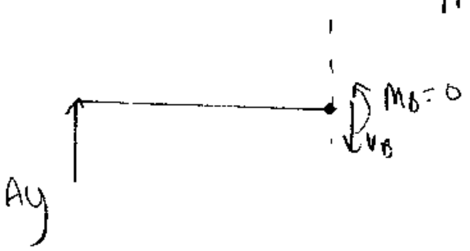


$\Sigma F_y = 0 = A_y - 30^k + C_y$

$A_y + C_y = 30^k$

$\circlearrowleft^{+} \Sigma M_C = 0 = M_C + 30(10) - A_y(20)$

$M_C = (A_y)20 - 300$

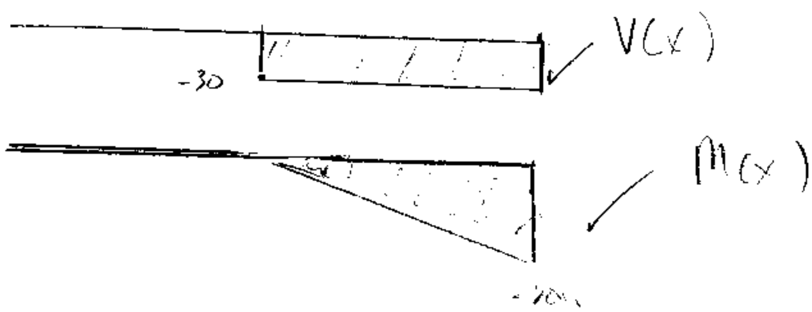


$\Sigma M_B = 0 = A_y(10)$

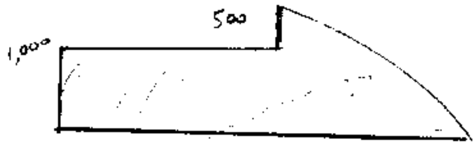
$A_y = 0$

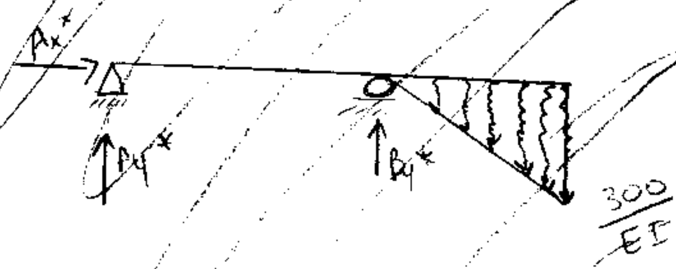
$C_y = 30^k$, pos ∴ ↑

$M_C = -300$ neg ∴



Conjugate Beam





SORRY

$\Sigma F_y = 0 = A_y^* + B_y^* - \left[\frac{300}{EI}\right][10][.5]$

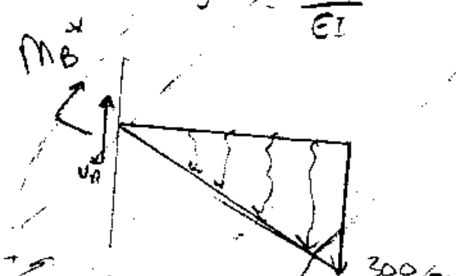
$A_y^* + B_y^* = \frac{1500}{EI}$

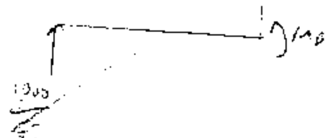
$+\circlearrowleft \Sigma M_B = 0 = A_y^*(10) + \left[\frac{300}{EI}\right][10][.5]\left[\frac{10 \cdot 2}{3}\right]$

$10A_y^* = -\frac{10,000}{EI}$

$A_y^* = -\frac{1000}{EI}$

$B_y^* = \frac{2500}{EI}$





$\Sigma M_B = 0 = M_B -$

$+\circlearrowleft \Sigma M_B = 0 = M_B^* + \frac{300}{EI}[10][.5]\left[\frac{10 \cdot 2}{3}\right]$

$M_B^* = -\frac{10,000}{EI}$ k·ft³

E = 4,000 ksi

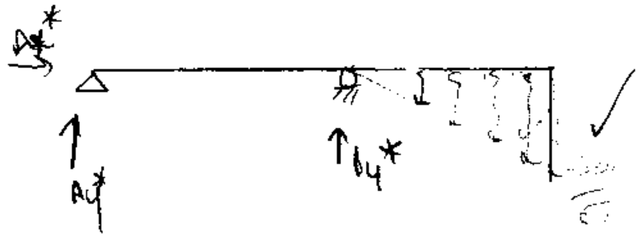
I = 3,000 in⁴

$$\frac{-10000 \text{ kft}^3 \left(\frac{12''}{1'}\right)^3}{(4,000)(3,000)}$$

$\boxed{y_B = -1.44 \text{ in}, \text{ neg} \therefore \downarrow}$

Conjugate beam loaded w/ the curvature

M/EI



$$\Sigma F_y = 0 = A_y^* + B_y^* - \frac{300}{EI}(10)(.5)$$

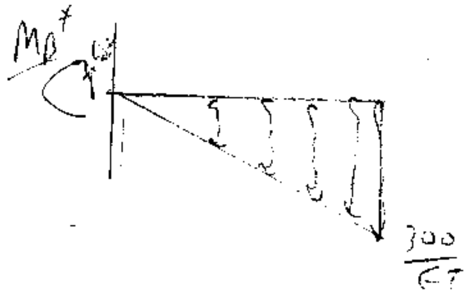
$$A_y^* + B_y^* = \frac{1500}{EI}$$

$$\Sigma M_B = 0 = A_y(10) + \frac{300[10][.5]}{EI}\left[\frac{2 \cdot 10}{3}\right]$$

$$10(A_y) =$$

$$A_y^* = \frac{-1000}{EI}$$

$$B_y^* = \frac{2500}{EI}$$



$$\Sigma M_B = 0 = M_B^* + \left[\frac{300}{EI}\right][10][.5]\left[\frac{10 \cdot 2}{3}\right]$$

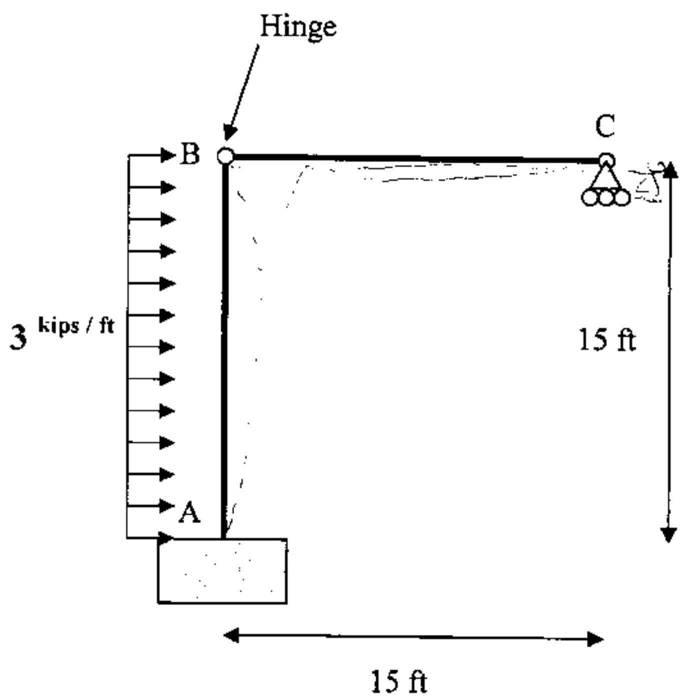
$$M_B^* = \frac{-10,000}{EI}$$

$$\boxed{M_B^* = -1.44 \text{ in}, \text{ ans} \therefore \downarrow}$$

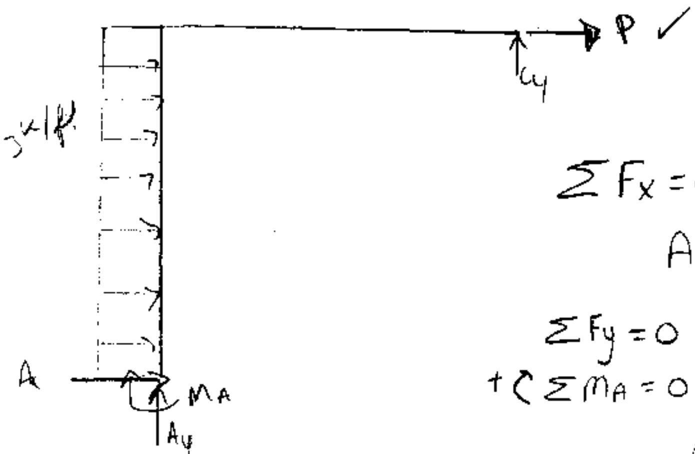
$$\frac{-10,000 \text{ k ft}^3 \left(\frac{12''}{1'}\right)^3}{(4000)(3000)}$$

33

5. [40 pts] Solve for the horizontal deflection at point C using either the virtual work method or Castigliano's second theorem. Be sure to clearly note which method of analysis you have chosen. Include shear and moment diagrams for the real force system shown below as part of your solution. Let E = 29,000 ksi and I = 2,000 in^4. EI is constant for the entire frame.



Castigliano's 2nd Thm will be used hopefully correctly!



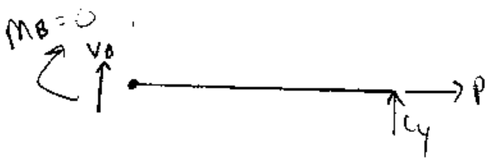
$\Sigma F_x = 0 = A_x + 3(15) + P$

$A_x = -45 - P$ ✓

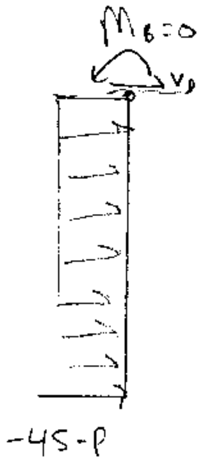
$\Sigma F_y = 0 = A_y + C_y = 0$

$+\circlearrowleft \Sigma M_A = 0 = 3(15)(7.5) + M_A - C_y(15) + P(15)$

$0 = 337.5 + M_A - 15C_y + P(15)$



$\Sigma M_B = 0 = C_y(15) \therefore C_y = 0$ ✓ ✓



$A_x = -45 - P$

$0 = 337.5 + M_a - 15C_y + P(15)$

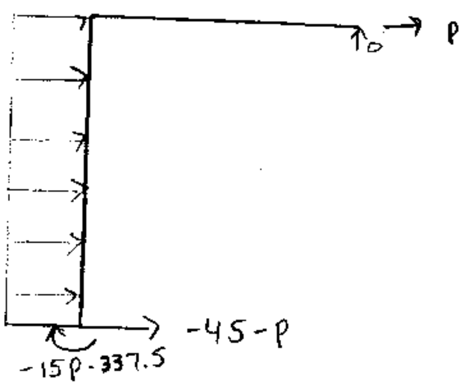
$A_y + C_y = 0$

$\circlearrowleft \sum M_B = 0 = (-45-P)(15) + 3(15)(7.5) - M_A$

$M_A = -675 - 15P + 337.5$

$M_A = -15P - 337.5$ ✓

$A_y = 0$ ✓
$C_y = 0$ ✓
$A_x = -45 - P$ ✓
$M_A = -15P - 337.5$ ✓



Segment	Origin	Limits	$M(x)$	$\frac{\partial M}{\partial P}$
AB	A	$0 \to 15$	$(45+P)x - 15P - 337.5$	$x - 15$ ✓

$\frac{1}{EI}\int_0^{15} [(45+\cancel{P})x - \cancel{15}P - 337.5][x-15]\,dx$

$\int_0^{15} (45x - 337.5)(x - 15)\,dx$

$\int_0^{15} 45x^2 - 675x - 337.5x + 5062.5$

$15x^3 - 506.25x^2 + 5062.5x \Big|_0^{15}$

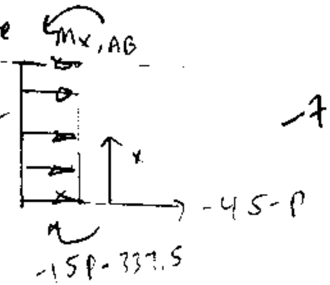
$50625 - 113906.25 + 75937.5$

$\frac{12656.25\ k\,ft^3}{EI}$

$= \frac{12{,}656.25\ k\cdot ft^3 \cdot \left(\frac{12''}{1'}\right)^3}{(29{,}000)(2{,}000)}$

$\boxed{= .3771\ in,\ pos. \rightarrow}$ ✗

member AB



incomplete FBD 3pt

-7.

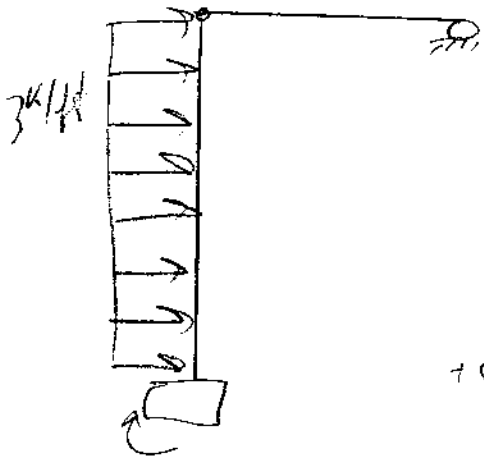
$\circlearrowleft \sum M_x = 0 = M_{x,AB} + (-45-P)(x) - (-15P - 337.5)$

$M_{x,AB} = -(-45-P)(x) + (-15P - 337.5)$

$M_{x,AB} = (45+P)(x) - 15P - 337.5$ ✗

$M_{x,BC} = 0$ ✓

Real Force System



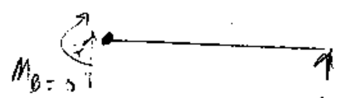
$\Sigma F_y = 0 = A_y + C_y = 0$

$\Sigma F_x = 0 = A_x + 3^k/ft\,(15)$

$A_x = -45^k$, neg $\therefore \leftarrow$

$+\circlearrowleft \Sigma M_A = 0 = 3(15)(7.5) - C_y(15) + M_A$

$M_A - 15C_y = -337.5$



$\Sigma M_B = 0 = C_y(15)$

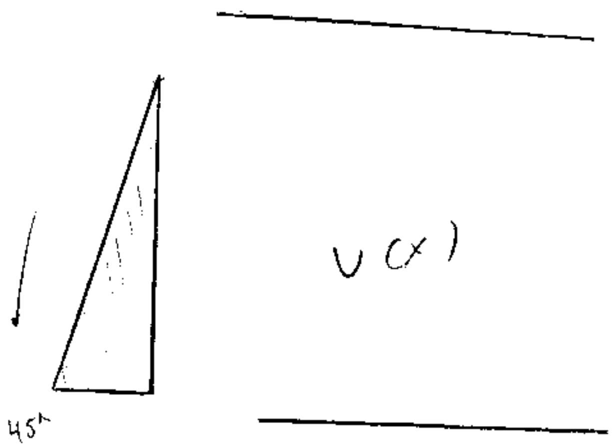
$C_y = 0$

$C_y = 0$

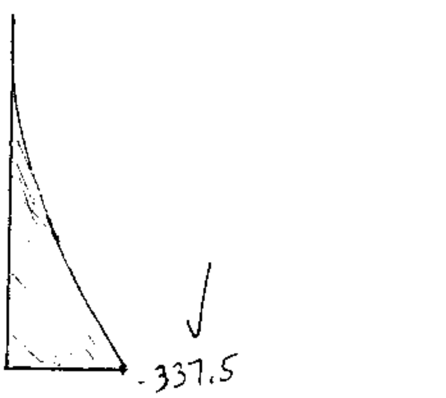
$A_y = 0$

$A_x = 45^k \leftarrow$

$M_A = -337.5$ neg $\therefore \circlearrowright$



V(x)



good

M(x)

$$\mathcal{U}_e = \mathcal{U}_i = \frac{1}{2} P\delta$$

$$\mathcal{U}_i = \frac{P^2 L}{2AE} \checkmark$$

79/100

CE 461 – Structural Analysis
EXAM No. 3

Name: ________

6

1. [6 pts] Identify three advantages to using indeterminate structures over determinate structures.

Indeterminate structures are

1. Redundant ✓
✓2. Have less bending stresses, therefore they can have smaller x-sections
~~3.~~ Have smaller displacements

0

2. [9 pts] Explain the presence of the ½ factor in the equation $U_e = \frac{1}{2} P\Delta$, which describes the amount of external work performed by a load which is applied to a structure gradually.

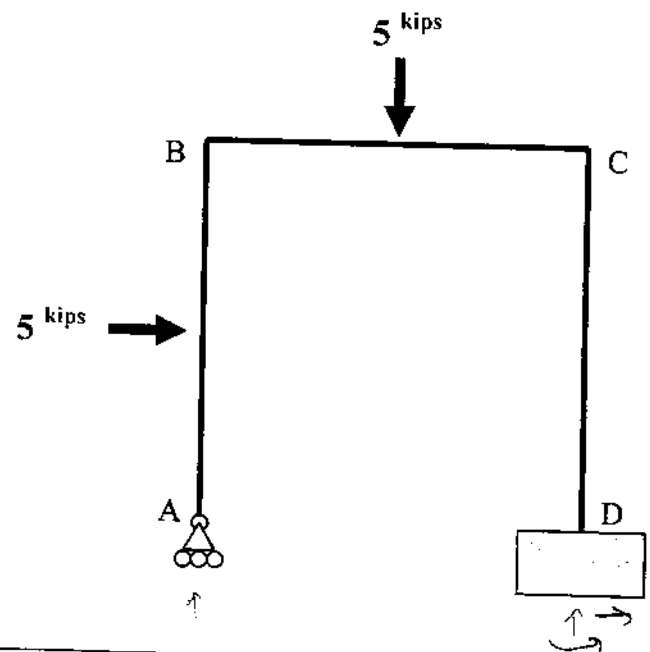
The $\frac{1}{2}$ factor is due to the fact that the shape of the deflected area resulting from the load at that point is in the shape of a triangle. Work is equal to the deflection shape of the structure. X

$U_e = \frac{1}{2} P . \Delta .$

(15) 3. [15 pts total] Given the following indeterminate frame to be analyzed using the force/flexibility method, with a final goal of solving for all support reactions:

a. [2 / 15 pts] Identify the degree of indeterminacy,
b. [3 / 15 pts] Identify the redundant(s) you would choose to solve for so that you could complete the analysis, and
c. [10 / 15 pts] Write the compatibility equation(s) to solve for those redundants.

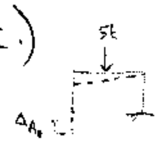
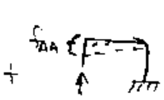
(Please note that you do not actually have to solve for the redundants to complete this problem.)



4 React., 3 eq. a.) Indeterminate to 1 degree ✓

b.) Only need 1 redundant, so choose A_y which would leave structure stable + determinate ✓

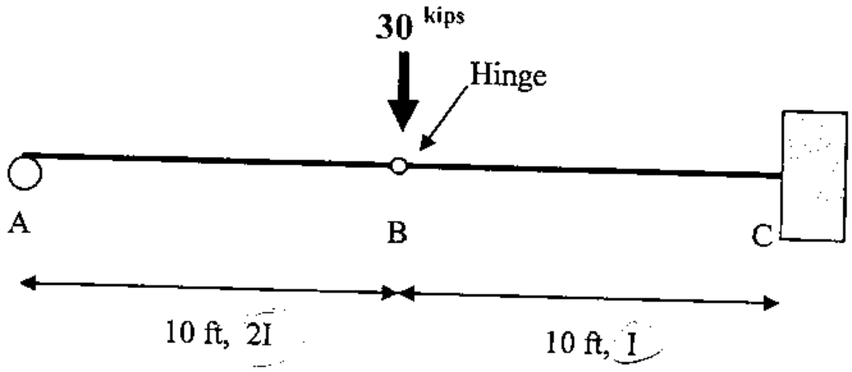
c.) Only need 1 compatibility equation

$0 =$  $+$  $\times A_y$

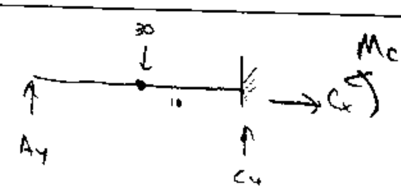
$$\boxed{0 = \Delta A_0 + f_{AA} \times A_y}$$ ✓

25

4. [30 pts] Solve for the vertical deflection at point B on the following beam. Use the conjugate beam method. Let E = 4,000 ksi and I = 3,000 in^4.



As whole



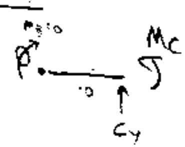
$+\uparrow \Sigma F_y = 0 = A_y - 30 + C_y$

$\Sigma F_x = 0 = C_x$

$\circlearrowleft + \Sigma M_c = 0 = -M_c - 30(10) + A_y(20)$

$-M_c - 300 + 20 A_y = 0$

To Right of hinge



$\circlearrowleft + \Sigma M_B = 0 = -M_c - 10 c_y$

$M_c = -10 C_y$

$-(-10 C_y) - 300 + 20 A_y$

Simultaneous Eq.
$10 C_y - 300 + 20 A_y$
$A_y - 30 + C_y$

$A_y = 30 - C_y$

$10 C_y - 300 + 20(30 - C_y)$

$10 C_y - 300 + 600 - 20 C_y$

$10 c_y = 300$

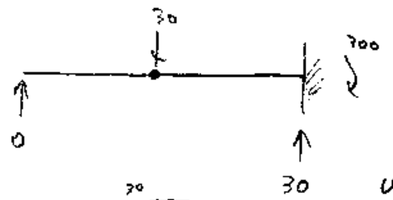
$C_y = 30$ pos. $\therefore \uparrow$

$+\uparrow \Sigma F_y = 0 = 30 + A_y - 30$

$A_y = 0$

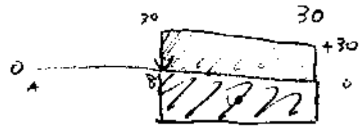
$-M_c - 300 + 20(A_y) = 0$

$M_c = -300$ neg $\therefore \circlearrowright$

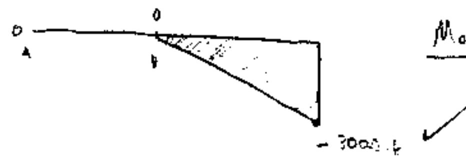


Real

upside-down (-2)



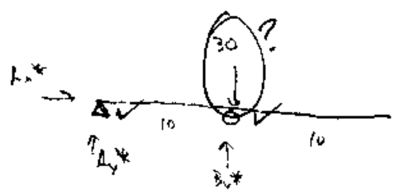
Shear



Moment

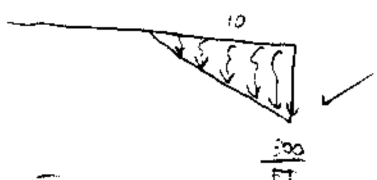
✓

Conjugate



Only conjugate loads should be applied to conjugate beam

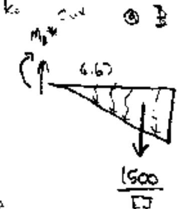
$A_x^* = 0$ (-3)



✓

For Δ @ B

Take cut @ B



$\circlearrowleft^+ \Sigma M_B = 0 = M_B^* + \frac{1500}{EI}(6.67)$

$M_B^* = -\frac{10{,}000}{EI}$

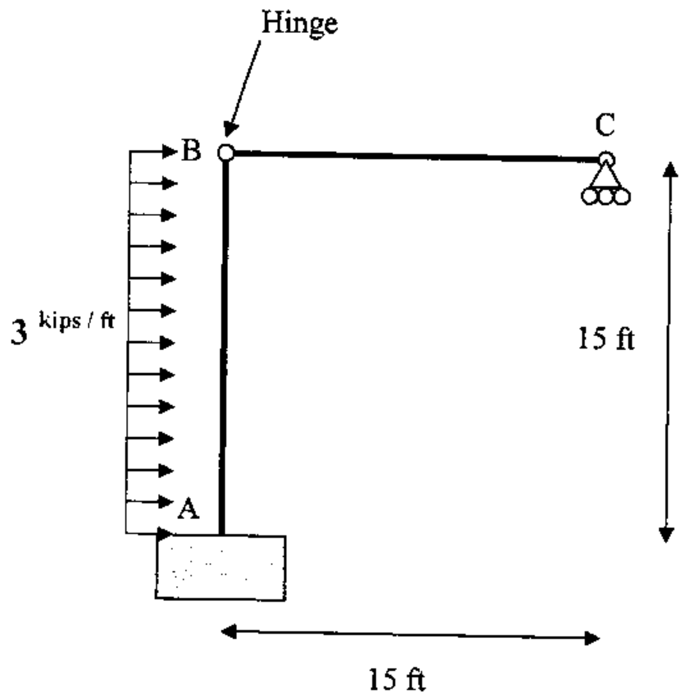
$\Delta_B = M_B^* = -\frac{10{,}000\,k\text{-}ft^3 \left(\frac{12\,in}{1\,ft}\right)^3}{(4000\,k/in^2)(3000\,in^4)} = -1.44$ ✓

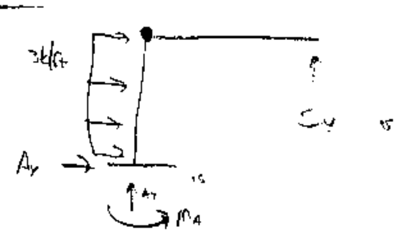
$\left(10{,}000\,k\text{-}ft^3\right)\left(\frac{(12\,in)^3}{1\,ft^3}\right)$

$\boxed{\Delta_B = -1.44\ \text{inches neg} \therefore \downarrow}$ ✓

33

5. [40 pts] Solve for the <u>horizontal</u> deflection at point C using either the virtual work method or Castigliano's second theorem. Be sure to clearly note which method of analysis you have chosen. Include shear and moment diagrams for the real force system shown below as part of your solution. Let E = 29,000 ksi and I = 2,000 in^4. EI is constant for the entire frame.





$+\uparrow \Sigma F_y = 0 = A_y + C_y$

$\rightarrow \Sigma F_x : 0 = (3)(15) + A_x$

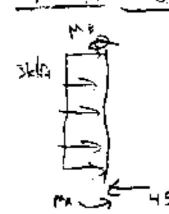
$A_x = -45$ neg $\therefore \leftarrow$

$\curvearrowright + M_A = 0 = -M_A + 3(15)\left(\frac{15}{2}\right) - C_y$

$= -M_A + 337.5 - 15C_y$

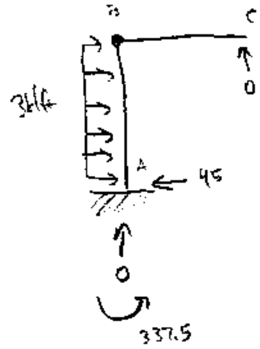
$C_y = 0$

$+\uparrow \Sigma F_y = 0 = \underline{A_y}$



$\curvearrowright \Sigma M_B = 0 = 45(15) - 3(15)(7.5) - M_A$

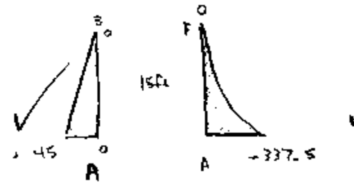
$\underline{M_A = 337.5}$



Shear & Moment

A-B

Shear Moment



$V_{max} = 45$ $M_{max} = -337.5$

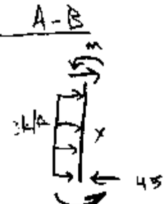
B-C

Shear Moment

$V = 0$ $M = 0$

Real Moments Equations

Section A-B



$\circlearrowleft \Sigma M = 0: \ -M - 3(x)\left(\frac{x}{2}\right) + 45x$

$M = -\frac{3x^2}{2} + 45x - 337.5 \quad 0 \le x \le 15$

337.5 Incomplete FBD -7

Section B-C

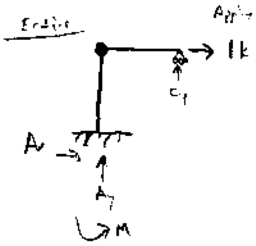
$M = 0 \quad 0 \le x \le 15$

☆ Using Virtual Work

Virtual Force System

Apply 1k → @C

Entire



$\uparrow \Sigma F_y = 0 = A_y + C_y$

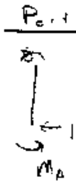
$\xrightarrow{+} \Sigma F_x = 0: \ A_x + 1$

$\circlearrowleft \Sigma M_A = 0: \ -M_A + 1(15)$

$M_A = 15$

$A_x = -1$ neg ∴ ←

Portion



$\circlearrowleft \Sigma M_B = 0 = -M_A + 1(15)$

$M_A = 15$ pos ∴ ↺

$A_y = 0$

$C_y = 0$

Virtual Moments

Section A-B

$$\circlearrowleft \Sigma M = 0 = -M - 15 + 1(x)$$

$$M = x - 15 \qquad 0 \le x \le 15$$

Section B-C

$$\circlearrowleft \Sigma M = 0 = M - 0(x)$$

$$M = 0 \qquad 0 \le x \le 15$$

Tabulate

Section	Length	Virtual m	Real M
A-B	15 ft	x-15	$-\frac{3x^2}{2} + 45x - 337.5$
B-C	15 ft	0	0

$$1\Delta = \frac{1}{EI}\int_0^{15} (x-15)\left(-\frac{3x^2}{2} + 45x\right)$$

$$1\Delta = \frac{1}{EI}\int_0^{15} \left(-1.5x^3 + 67.5x^2 - 675x\right) dx$$

$$1\Delta = \frac{1}{EI}\left(-0.375x^4 + 22.5x^3 - 337.5x^2\right)\Big|_0^{15}$$

$$-18{,}984.4 \qquad 75{,}937.5 \qquad - \quad 75{,}937.5$$

$$\Delta = -\frac{18{,}984.4}{EI}$$

$$= \frac{-18{,}984.4\,k\,ft^3 \left(\frac{(12\,in)^3}{ft^3}\right)}{(29{,}000\,ksi)(2000\,in^4)}$$

$$\boxed{\Delta = -0.566 \text{ inches}}$$

×

Bonus (5 pts): Derive the following equation, which describes the amount of internal strain energy that develops in an axially loaded member when the member is gradually loaded with a force, <u>in three steps or less</u>.

$$\boxed{U_i = \frac{N^2 L}{2AE}}$$

76/100

CE 461 – Structural Analysis
EXAM No. 3

Name:_

6

1. [6 pts] Identify three advantages to using indeterminate structures over determinate structures.

- REDUNDANCY ✓
- SMALLER STRESSES ✓
- SMALLER DEFLECTIONS ✓

Good

0

2. [9 pts] Explain the presence of the ½ factor in the equation $U_e = \frac{1}{2}P\Delta$, which describes the amount of external work performed by a load which is applied to a structure gradually.

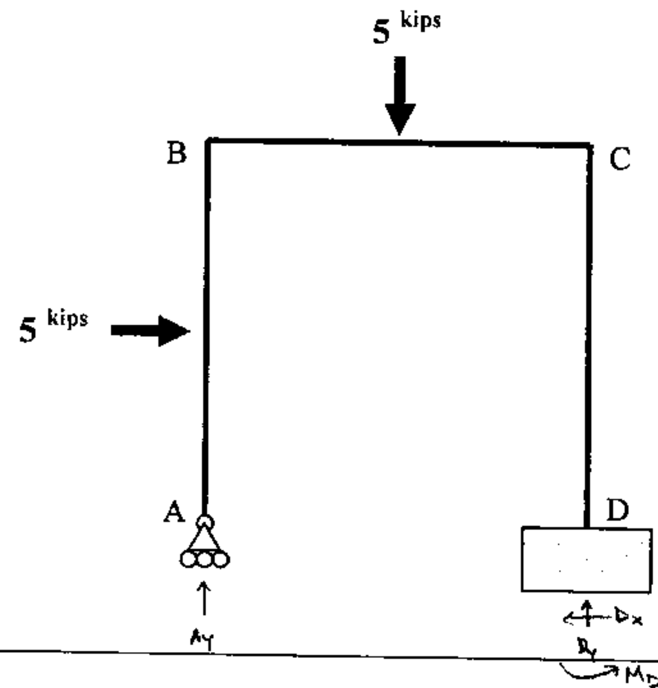
SINCE THIS IS 9 POINTS I REALLY WISH I COULD

SORRY

(15) 3. [15 pts total] Given the following indeterminate frame to be analyzed using the force/flexibility method, with a final goal of solving for all support reactions:

2 a. [2 / 15 pts] Identify the degree of indeterminacy,
3 b. [3 / 15 pts] Identify the redundant(s) you would choose to solve for so that you could complete the analysis, and
10 c. [10 / 15 pts] Write the compatibility equation(s) to solve for those redundants.

(Please note that you do not actually have to solve for the redundants to complete this problem.)



a) UNKNOWNS 4 EQNS 3 ∴ 1° INDET ✓

b) M_D WOULD BE CHOSEN AS A REDUNDANT MAKING POINT D A PIN AND ALLOWING THE FRAME TO REMAIN STABLE. ✓ Good

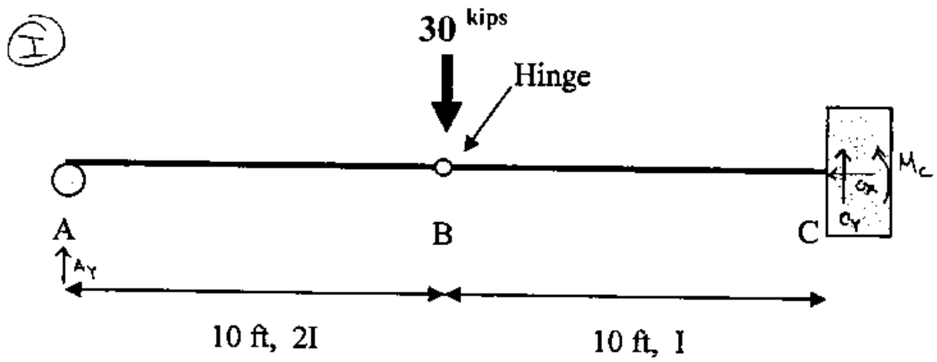
c) $\Delta\theta_D = 0 = \Delta\theta_{DO} + f_{DD} \times M_D$ ✓

$\Delta\theta_{DO}$ = ANGLE OF DEFLECTION AT POINT D OF PRIMARY BEAM DUE TO REAL LOADING (5k LOADS) ✓

f_{DD} = ANGLE OF DEFLECTION AT POINT D OF PRIMARY BEAM DUE TO UNIT COUPLE PLACED AT D (POINT OF INTEREST) ✓

30

4. [30 pts] Solve for the vertical deflection at point B on the following beam. Use the conjugate beam method. Let E = 4,000 ksi and I = 3,000 in^4.



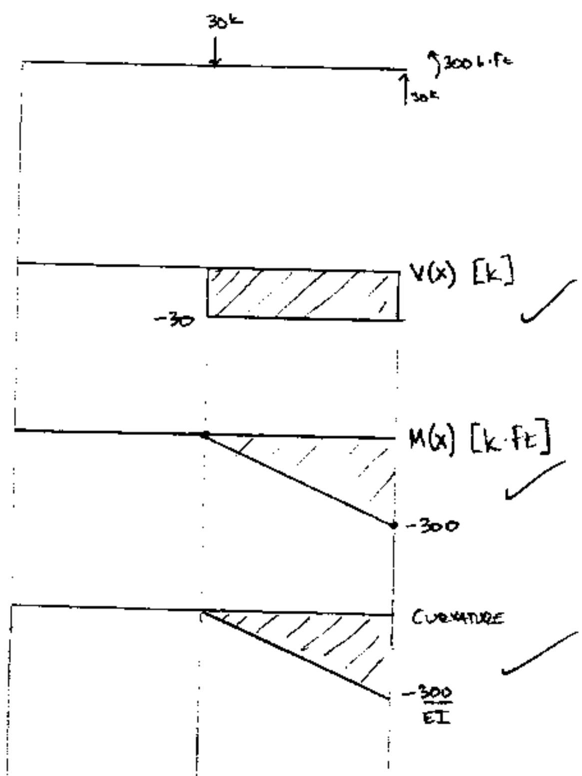
$\rightarrow + \sum F_{X(AC)} = 0 = -C_X$ $\qquad C_X = 0$

$\circlearrowleft + \sum M_{B(AB)} = 0 = A_Y\,(10ft)$ $\qquad A_Y = 0$

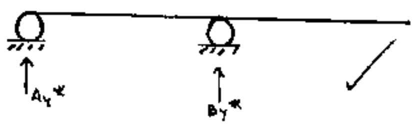
$\uparrow + \sum F_{Y(AC)} = 0 = \cancel{A_Y}^{0} + C_Y - 30^k$ $\qquad C_Y = 30^k$, pos ∴ ↑

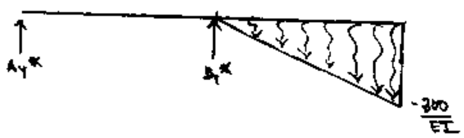
$\circlearrowleft + \sum M_{B(BC)} = 0 = (C_Y)(10ft) + M_C$

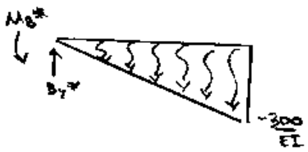
$0 = (30^k)(10ft) + M_C$ $\qquad M_C = -300\ k\cdot ft$, neg ∴ ↻



C.B.







$$\circlearrowleft + \Sigma M_B = 0 = M_B^* - \left(\frac{300}{EI}\right)(10\,\text{ft})\left(\tfrac{1}{2}\right)\left(\frac{20}{3}\,\text{ft}\right)$$

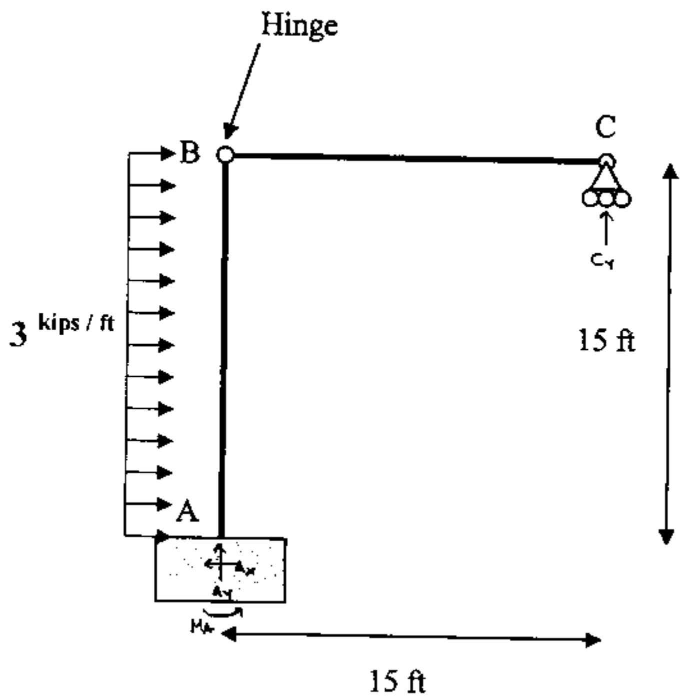
$$M_B^* = \frac{10000}{EI}$$

$$M_B^* = \frac{10000}{(4{,}000\ \text{k/in}^2)\left(\frac{12\,\text{in}}{1\,\text{ft}}\right)^2 (3{,}000\ \text{in}^4)\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)^4}$$

$$\boxed{M_B^* = 0.12\ \text{ft} \quad \text{or} \quad 1.44\ \text{in}\ \text{, pos} \therefore \downarrow}$$

25

5. [40 pts] Solve for the horizontal deflection at point C using either the virtual work method or Castigliano's second theorem. Be sure to clearly note which method of analysis you have chosen. Include shear and moment diagrams for the real force system shown below as part of your solution. Let E = 29,000 ksi and I = 2,000 in^4. EI is constant for the entire frame.



VIRTUAL WORK

$\circlearrowleft + \Sigma M_{B\,(BC)} = 0 = C_Y(15ft)$ $\quad C_Y = 0$

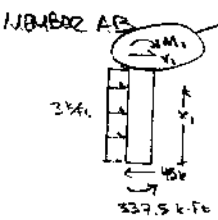
$\uparrow + \Sigma F_{Y\,(ABC)} = 0 = A_Y + \cancel{C_Y}^{0}$ $\quad A_Y = 0$

$\rightarrow + \Sigma F_{X\,(ABC)} = 0 = (3\,k/ft)(15ft) - A_X$ $\quad A_X = 45k$

$\circlearrowleft + \Sigma M_{A\,(AB)} = 0 = M_A - (3k/ft)(15ft)(\frac{15}{2}ft)$ $\quad M_A = 337.5\ k\cdot ft$

MEMBER AB

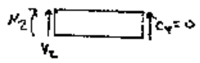
Your moment is drawn here as neg^ve beam sign convention...



$\circlearrowleft + \Sigma M_x = 0 = -M_1 + (3k/ft)(x)(x/2) - (45k)(x) + 337.5\ k\cdot ft$

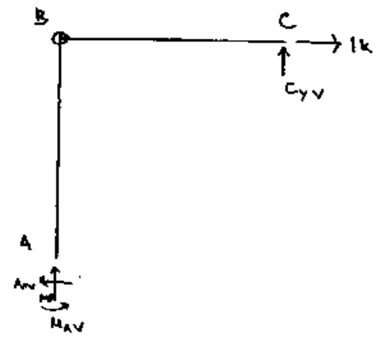
$$M_1 = \frac{3x^2}{2} - 45x + 337.5$$

MEMBER BC



$\circlearrowleft + \Sigma M_x = 0 = -M_2 + \cancel{C_Y}^{0}(x)$

$M_2 = 0$



$\xrightarrow{+} \Sigma F_{X\,(ABC)} = 1k - A_{XV}$ $\qquad A_{XV} = 1k$

$\circlearrowleft + \Sigma M_{B\,(BC)} = 0 = C_{YV}(15ft)$ $\qquad C_{YV} = 0$

$\uparrow + \Sigma F_{Y\,(ABC)} = 0 = A_{YV} + \cancel{C_{YV}}^{0}$ $\qquad A_{YV} = 0$

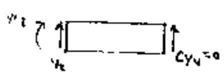
$\circlearrowleft + \Sigma M_{A\,(ABC)} = 0 = M_{AV} - (1k)(15ft)$ $\qquad M_{AV} = 15\ k\cdot ft$

MEMBER AB

$\circlearrowleft + \Sigma M_x = 0 = 15\ k\cdot ft - (1k)(x) - m_1$

$m_1 = 15 - x$

MEMBER BC



$\circlearrowleft + \Sigma M_x = 0 = \cancel{C_{YV}}(x) - m_2$

$m_2 = 0$

$(15-x)\left(\frac{3x^2}{2} - 45x + \frac{675}{2}\right)$

$\cancel{\frac{45x^2}{2}} - \cancel{675}x + \frac{10125}{2} - \frac{3x^3}{2} + 45\cancel{x^2} - \frac{675}{2}$

$\frac{45x^2}{2} - 675x + \frac{10125}{2} \ldots + 45x^2 - \frac{675}{2}$

$$1(\Delta) = \Sigma\int_0^L \frac{m_V M}{EI}\,dx$$

$$= \int_0^{15} \frac{(15-x)\left(\frac{3x^2}{2} - 45x + 337.5\right)}{EI}\,dx + \int_0^{15} \frac{(0)(0)\,dx}{EI}$$

$$= \frac{1}{EI}\int_0^{15} -\frac{3x^3}{2} + \frac{135x^2}{2} - \frac{2025x}{2} + \frac{10125}{2}$$

$$= \frac{1}{EI}\left. -\frac{3x^4}{8} + \frac{135x^3}{6} - \frac{2025x^2}{4} + \frac{10125}{2}x\ \right|_0^{15}$$

$$1(\Delta) = \frac{18984.375}{EI}$$

$$= \frac{18984.375}{\left(29{,}000\ \frac{k}{in^2}\right)\left(\frac{12in}{1ft}\right)^2(2000in^4)\left(\frac{1ft}{12in}\right)^4}$$

$$\boxed{\Delta_{C\,(HORIZONTAL)} = 0.0471\ ft \quad or \quad 0.566\ in\ ,\ pos\ \therefore \longrightarrow}$$

Shear & moment diagrams for RFS?

−15

Bonus (5 pts): Derive the following equation, which describes the amount of internal strain energy that develops in an axially loaded member when the member is gradually loaded with a force, <u>in three steps or less</u>.

$$\boxed{U_i = \frac{N^2 L}{2AE}}$$

89.5/100

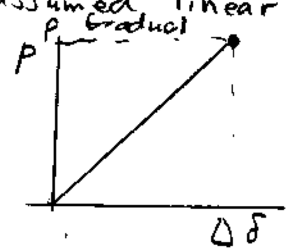
CE 461 – Structural Analysis **Name**
EXAM No. 3

(6) 1. [6 pts] Identify three advantages to using indeterminate structures over determinate structures.

Indeterminate structures have redundancy – one part may fail without the entire structure becoming unstable. An indeterminate structure will have a smaller internal moment and deflection than determinate structures. Indeterminate structures are cheaper and easier to fabricate. not necessarily!

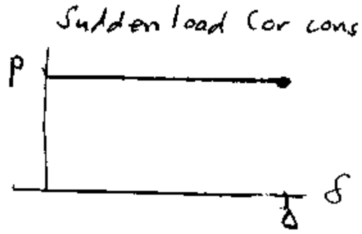
(9) 2. [9 pts] Explain the presence of the ½ factor in the equation $U_e = \frac{1}{2}P\Delta$, which describes the amount of external work performed by a load which is applied to a structure gradually.

Energy must be conserved. For gradual loading, an incremental increase in the load ∂P causes a change in deflection $\partial\delta$. Since the material is assumed linear elastic, the plot of load vs. deflection will be linear.



Work = $\int P\,\partial\delta$. This is equal to the area under the P vs. δ curve, or ½ PΔ.

In contrast a sudden loading has the following graph:



The area under this curve is simply PΔ, as the load "moves" with the structure.

15

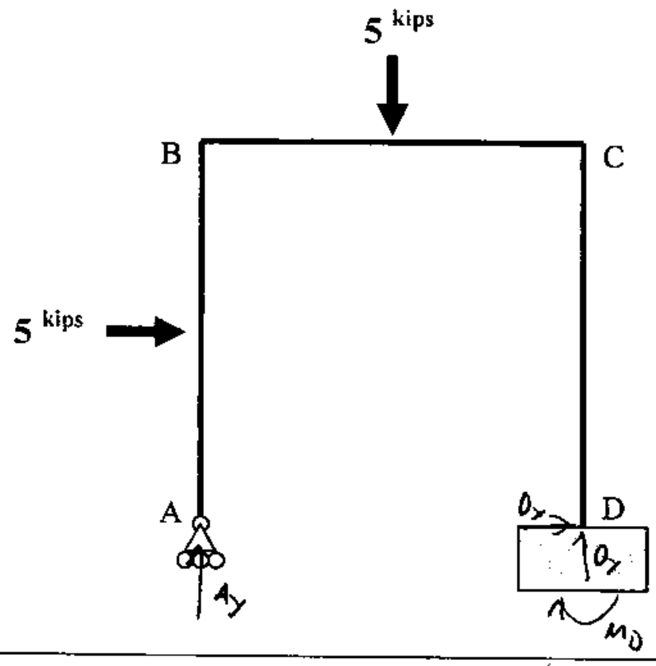
3. [15 pts total] Given the following indeterminate frame to be analyzed using the force/flexibility method, with a final goal of solving for all support reactions:

2 a. [2 / 15 pts] Identify the degree of indeterminacy,

3 b. [3 / 15 pts] Identify the redundant(s) you would choose to solve for so that you could complete the analysis, and

10 c. [10 / 15 pts] Write the compatibility equation(s) to solve for those redundants.

(Please note that you do not actually have to solve for the redundants to complete this problem.)



a) 4 unknowns > 3 equations, ∴ 1° indeterminate ✓

b) Remove A_y (solve for A_y) ✓

c) Require $A_y = 0$

$0 = \Delta A_0 + f_{AA} A_y$, where good

ΔA_0 = Deflection (vertical) at A on the primary structure due to real loads (Fig. A) ✓

f_{AA} = Deflection (vert.) at A on primary structure due to 1k vert. load @ A (Fig. B)

A_y = Reaction at A ✓

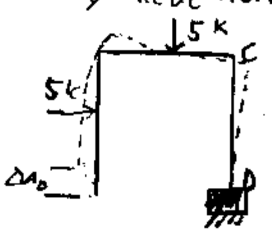


Fig. A

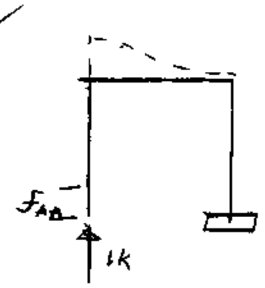
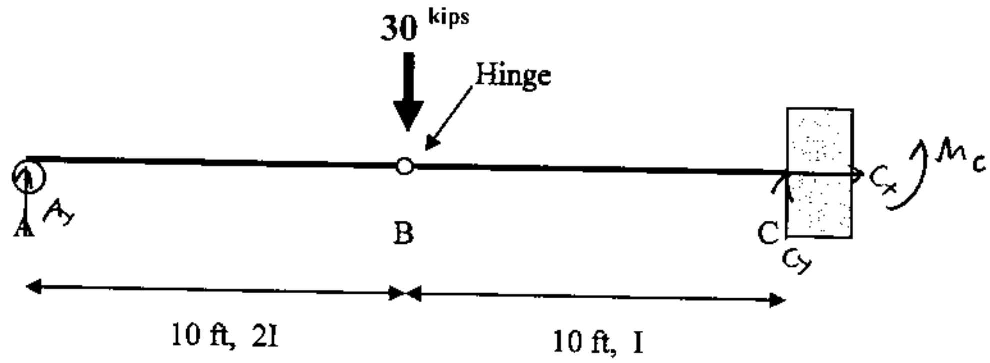


Fig. B

$\times A_y$

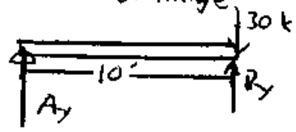
(29) 4. [30 pts] Solve for the vertical deflection at point B on the following beam. Use the conjugate beam method. Let E = 4,000 ksi and I = 3,000 in^4.



1) Solve for rxns:

$\Sigma F_x = 0;\ C_x = 0$

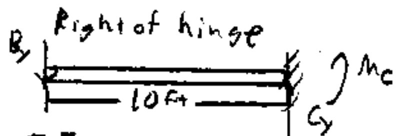
Left of hinge



$\Sigma M_B = 0;\ A_y = 0$

$\Sigma F_y = 0;\ B_y = 30\text{ k}$

Right of hinge



$\Sigma F_y = 0;\ C_y = 30\text{ k}$

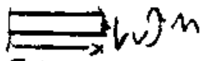
$\Sigma M_c = 0$

$30\text{ k} \cdot 10' + M_c = 0$

$M_c = -300\text{ ft}\cdot\text{k}$

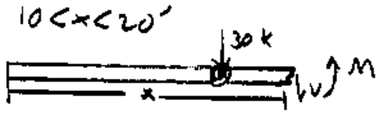
2) Find Moment as a function of x.

$0 < x < 10'$



$\Sigma M_x = 0,\ M = 0$

$10 < x < 20'$

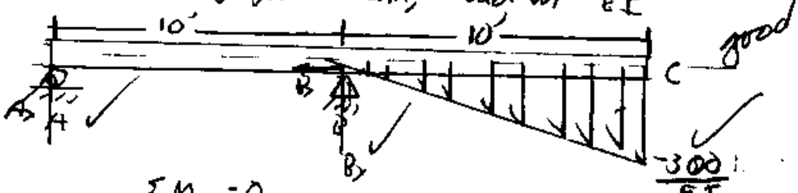


$\Sigma M_x = 0$

$M + 30(x-10) = 0$

$M = 300 - 30x$

3) Draw conjugate beam, load w/ $\frac{M}{EI}$



good

$\Sigma M_A = 0$

$B_y \cdot 10 - (\tfrac{1}{2} \cdot 10 \cdot \tfrac{300}{EI}) \cdot (10 + \tfrac{2}{3}(10)) = 0$

$B_y = \frac{2500\text{k}}{EI}$

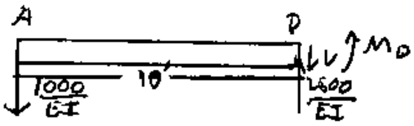
$\Sigma F_y = 0$

$\frac{2500}{EI} + A_y - \tfrac{1}{2} \cdot 10 \cdot \frac{300}{EI} = 0$

$A_y = -\frac{1000\text{k}}{EI}$

We want deflection at B ∴ Find M'_B on the conjugate beam

Break beam at B



$\Sigma M_B = 0$

$\frac{1000}{EI} \cdot 10' + M'_B = 0$

$M'_B = \frac{10{,}000}{EI} \frac{ft^3 \cdot k}{k \cdot in^2}$

Convert EI to $k \cdot ft^2$

$E = 4000$ ksi

$I = 3000\ in^4$ (2I section had no load)

$EI = 1.2 \cdot 10^7\ k \cdot in^2 \cdot \frac{1ft^2}{144 in^2} = 83{,}333.33\ k \cdot ft^2$

$M'_B = v_B = \frac{10{,}000}{83{,}333} = 0.12 ft = 1.44 in$

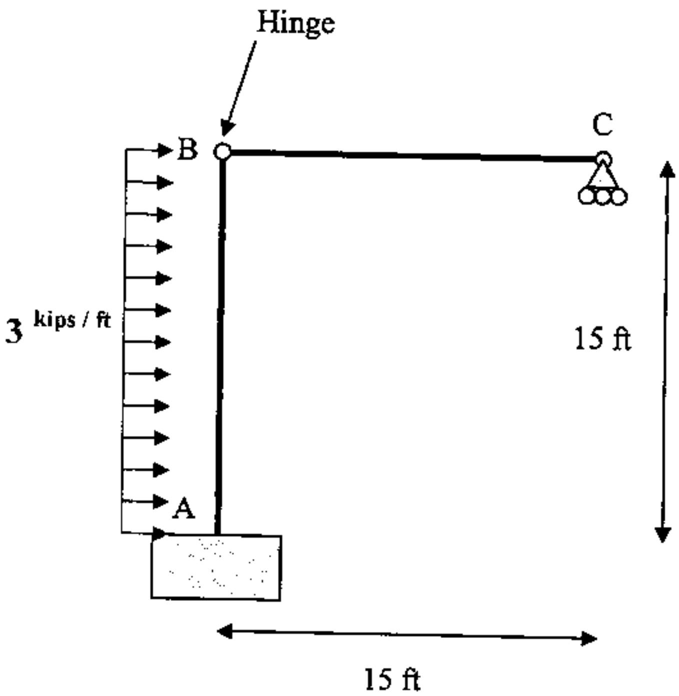
$v_B = 1.44 in$ ✓

which direction?

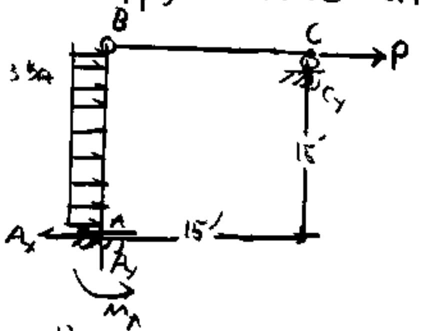
-1

30.5

5. [40 pts] Solve for the horizontal deflection at point C using either the virtual work method or Castigliano's second theorem. Be sure to clearly note which method of analysis you have chosen. Include shear and moment diagrams for the real force system shown below as part of your solution. Let E = 29,000 ksi and I = 2,000 in^4. EI is constant for the entire frame.

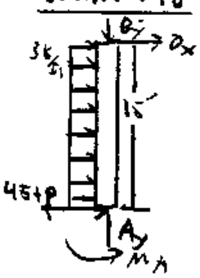


Castiglianos 2nd Theorm
Apply variable horizontal Force at C



$\Sigma F_x = 0$
$\underline{A_x = (45 + P)\ K}$

<u>Beam AB</u>

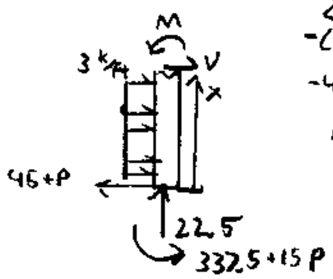


B cannot support a moment
$\Sigma F_x = 0$
$-(45+P) + 45 + B_x = 0$
$B_x = P$
$\Sigma M_A = 0$
$-15 \cdot P + -45 \cdot 7.5 + M_A = 0$
$\underline{M_A = 337.5 + 15P}$ ✓

<u>Overall</u>
$\Sigma M_A = 0$ Not OK
$337.5 + 15P + C_y \cdot 15 - P \cdot 15 = 0$
$\underline{C_y = -22.5\ kN}$ X (-2)
$\Sigma F_y = 0$
$\underline{A_y = 22.5\ kN}$ X

Find internal moments

AB



$\Sigma M_x = 0$

$-(45+P)x + 3x \cdot \frac{x}{2} + 337.5 + 15P + M = 0$

$-45x - Px + \frac{3}{2}x^2 + 337.5 + 15P + M = 0$

$M = 45x - \frac{3}{2}x^2 - P(15-x) - 337.5$

$\frac{\partial M}{\partial P} = -15 + x$

$M(P=0) = 45x - \frac{3}{2}x^2 - 337.5$

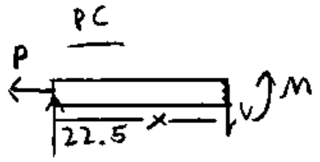
Check: $M(B) = 0$

$\Sigma F_x = 0$

$V = 45 + P - 3x$

$V(P=0) = 45 - 3x$

BC



$\Sigma F_y = 0$

$V = 22.5$

$\Sigma M_x = 0$

$M = 22.5x$

$\frac{\partial M}{\partial P} = 0$

$M(P=0) = 22.5x$

$\Delta C_x = \Sigma \int \frac{\partial M}{\partial P} \cdot \frac{M}{EI}\, \partial x$

$\Delta C_x = \frac{1}{EI}\left[\int_0^{15} (-15+x)(45x - \frac{3}{2}x^2 - 337.5)dx + \int_0^{15} 0 \cdot \cancel{22.5x}^{\,0}\, dx\right]$

$= \frac{1}{EI}\int_0^{15}(-675x + 22.5x^2 + 5062.5 + 45x^2 - \frac{3}{2}x^3 - 337.5x)dx$

$= \frac{1}{EI}\int_0^{15}(-\frac{3}{2}x^3 + 67.5x^2 - 1012.5x + 5062.5)dx$

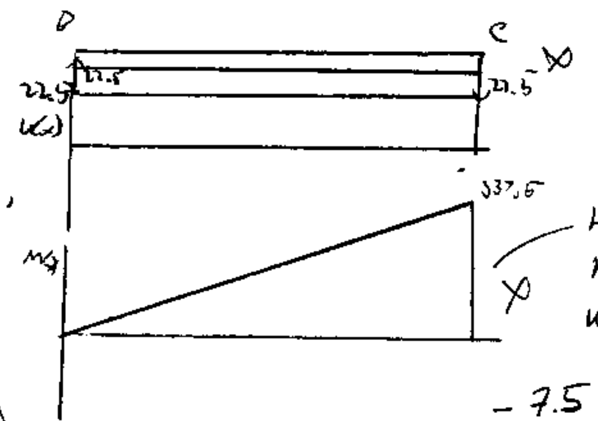
$= \frac{1}{EI}\left[-\frac{3}{8}x^4 + \frac{67.5}{3}x^3 - 506.25x^2 + 5062.5x\right]_0^{15}$

$\Delta C_x = \frac{1}{EI}(18984.375)$

$EI = 5.8 \cdot 10^7\ k \cdot in^2 = 4.027 \cdot 10^5\ k\,ft^2$

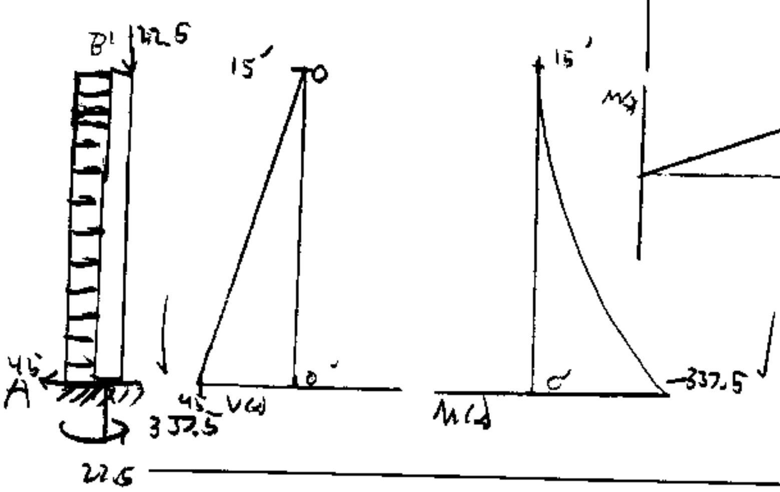
$\Delta C_x = 0.0471\ ft = \underline{0.566\ in}$

$\Delta E_x = 0.566\ in$ ✓



How could there be a moment at point C, which is a hinge?

- 7.5



r

0 Bonus (5 pts): Derive the following equation, which describes the amount of internal strain energy that develops in an axially loaded member when the member is gradually loaded with a force, in three steps or less.

$$\boxed{U_i = \frac{N^2 L}{2AE}}$$

δ ~~Stress~~ $= \frac{NL}{AE}$

Total energy $= \int \cancel{\sigma}\, dx$ (N is a function of x - gradual load)

$U_i = \int \frac{NL}{AE} = \frac{N^2 L}{2AE}$

Internal Work (Strain Energy) for Axial Forces $U_i = \dfrac{N^2 L}{2AE}$

Internal Work (Strain Energy) for Bending $U_i = \int_0^L \dfrac{M^2 dx}{2EI}$

Internal Work (Strain Energy) for Shear $U_i = \int_0^L \left(\dfrac{V^2}{2GA_v}\right) dx$, where $A_v = A/K$

K = 1.2 for rectangles
K = 10/9 for circular cross-sections
K = approx. 1 for I-beams, where A is the area of the web
K = 2 for tubes

Method of Virtual Work Applied to Trusses $1(\Delta) = \sum n_v \left(\dfrac{NL}{AE}\right)$

Method of Virtual Work Applied to Beams

$$\begin{cases} 1(\Delta) = \left[\int_0^L \dfrac{m_v M}{EI} dx\right] + \left[\int_0^L \dfrac{v_v V}{GA_v} dx\right] \\ 1(\theta) = \left[\int_0^L \dfrac{m_v M}{EI} dx\right] + \left[\int_0^L \dfrac{v_v V}{GA_v} dx\right] \end{cases}$$

Method of Virtual Work Applied to Frames

$$\begin{cases} 1(\Delta) = \left[\sum n_v \left(\dfrac{NL}{AE}\right)\right] + \left[\sum \int_0^L \dfrac{m_v M}{EI} dx\right] + \left[\sum \int_0^L \dfrac{v_v V}{GA_v} dx\right] \\ 1(\theta) = \left[\sum n_v \left(\dfrac{NL}{AE}\right)\right] + \left[\sum \int_0^L \dfrac{m_v M}{EI} dx\right] + \left[\sum \int_0^L \dfrac{v_v V}{GA_v} dx\right] \end{cases}$$

Castigliano's Second Theorem Applied to Trusses $\Delta = \sum \left(\dfrac{\partial N}{\partial P}\right) \dfrac{NL}{AE}$

Castigliano's Second Theorem Applied to Beams

$$\begin{cases} \Delta = \left[\int_0^L \left(\dfrac{\partial M}{\partial P}\right) \dfrac{M}{EI} dx\right] + \left[\int_0^L \left(\dfrac{\partial V}{\partial P}\right) \dfrac{V}{GA_v} dx\right] \\ \theta = \left[\int_0^L \left(\dfrac{\partial M}{\partial \bar{M}}\right) \dfrac{M}{EI} dx\right] + \left[\int_0^L \left(\dfrac{\partial V}{\partial \bar{M}}\right) \dfrac{V}{GA_v} dx\right] \end{cases}$$

Castigliano's Second Theorem Applied to Frames

$$\begin{cases} \Delta = \left[\sum \left(\dfrac{\partial N}{\partial P}\right) \dfrac{NL}{AE}\right] + \left[\sum \int_0^L \left(\dfrac{\partial M}{\partial P}\right) \dfrac{M}{EI} dx\right] + \left[\int_0^L \left(\dfrac{\partial V}{\partial P}\right) \dfrac{V}{GA_v} dx\right] \\ \theta = \left[\sum \left(\dfrac{\partial N}{\partial \bar{M}}\right) \dfrac{NL}{AE}\right] + \left[\sum \int_0^L \left(\dfrac{\partial M}{\partial \bar{M}}\right) \dfrac{M}{EI} dx\right] + \left[\int_0^L \left(\dfrac{\partial V}{\partial \bar{M}}\right) \dfrac{V}{GA_v} dx\right] \end{cases}$$

85.5/100

CE 461 – Structural Analysis
EXAM No. 3

Name:_

11/6/06

6

1. [6 pts] Identify three advantages to using indeterminate structures over determinate structures.

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1. Redundancy ✓
2. Smaller deflections ✓
3. Smaller moments (internal) ✓

0

2. [9 pts] Explain the presence of the ½ factor in the equation $U_e = \frac{1}{2}P\Delta$, which describes the amount of external work performed by a load which is applied to a structure gradually.

Work = Force x distance

the one half factor stems from from

$\frac{1}{2}$ kE in the energy balance equation

↳ kinetic energy $\frac{1}{2}kE = \frac{1}{2}U_e = \frac{1}{2}P\Delta$ X

15

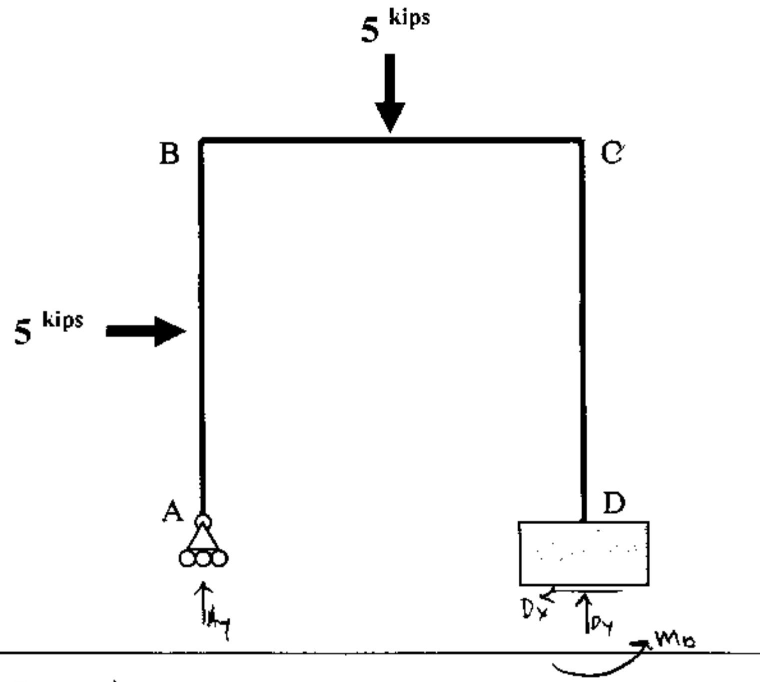
3. [15 pts total] Given the following indeterminate frame to be analyzed using the force/flexibility method, with a final goal of solving for all support reactions:

2 a. [2 / 15 pts] Identify the degree of indeterminacy,

3 b. [3 / 15 pts] Identify the redundant(s) you would choose to solve for so that you could complete the analysis, and

10 c. [10 / 15 pts] Write the compatibility equation(s) to solve for those redundants.

(Please note that you do not actually have to solve for the redundants to complete this problem.)

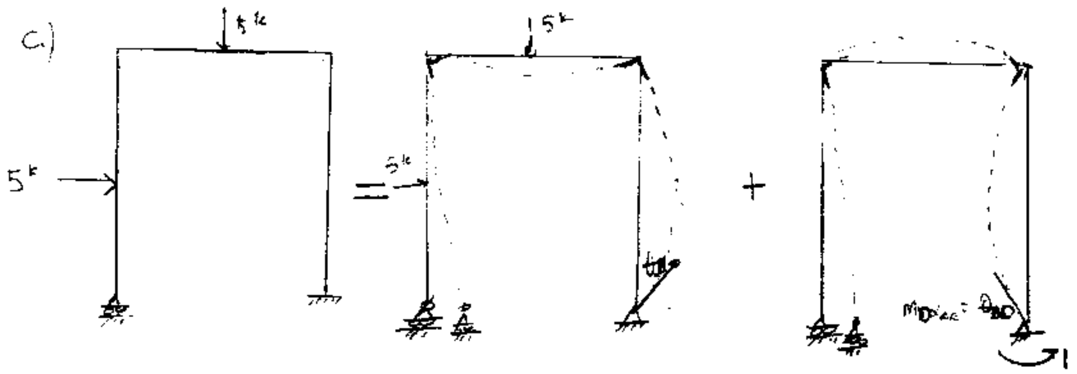


a.) 4 unknowns > 3 equations of equil. + 0 equations of condition

=> stable ∴ Indeterminate 1° ✓

✓ b.) I would remove the moment Reaction at D to complete the analysis. The removal of M_D leaves a stable, determinate structure to analyze OK

c.)



✓

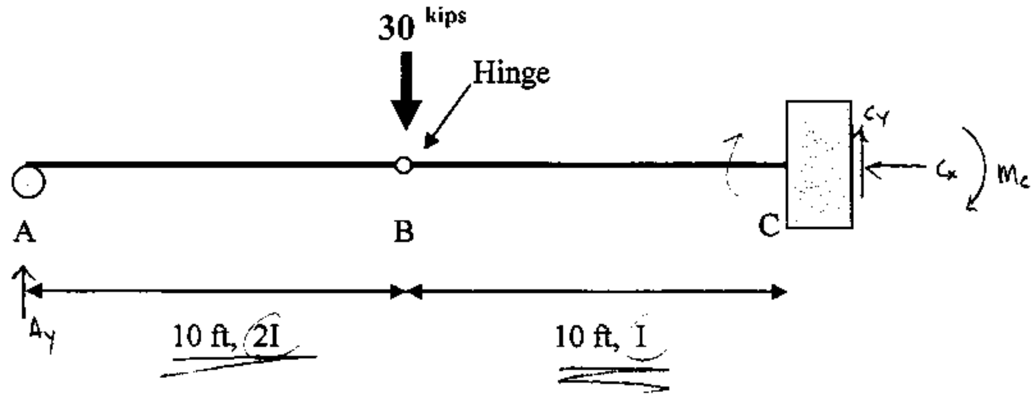
$\theta_A = 0 = \theta_{D0} + M_D \alpha_{DD}$

$0 = \theta_{D0} + M_D \alpha_{DD}$

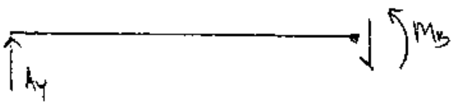
good

29

4. [30 pts] Solve for the vertical deflection at point B on the following beam. Use the conjugate beam method. Let E = 4,000 ksi and I = 3,000 in^4.



$\Sigma F_x = 0 = C_x$



using equation of condition

$\Sigma M_B = 0 = A_y(10)$

$A_y = 0$

analyzing the entire beam:

$\Sigma F_y = 0 = A_y - 30 + C_y$

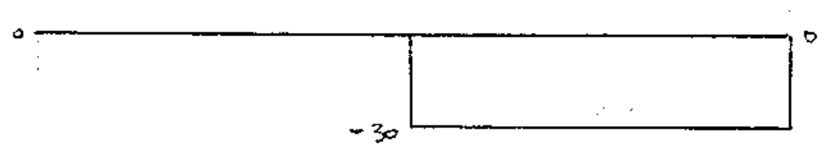
$0 = 0 - 30 + C_y$

$C_y = 30^k$

$\Sigma M_C = 0 = -M_c + 30(10) - A_y(20)$

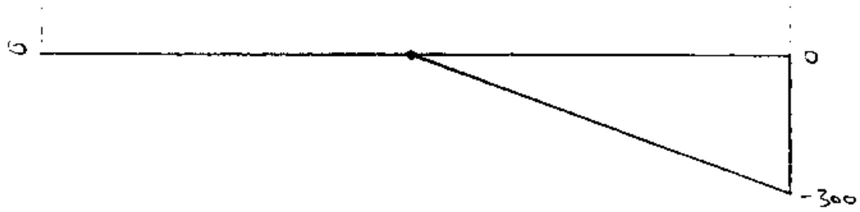
$M_c = 300$ ft·k

V(x) [kip]



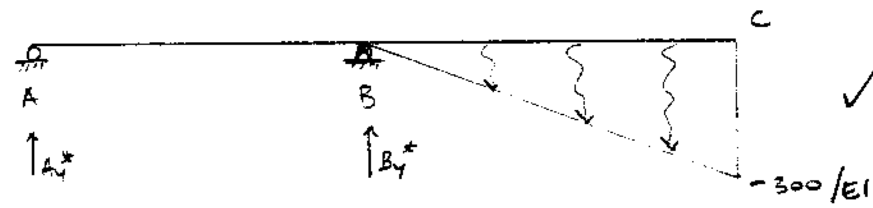
✓

M(x) [ft·kip]



✓

Conjugate Beam Loaded w/ Moment diagram of the real beam - Find Y_B



✓

$Y_B = M_B^*$

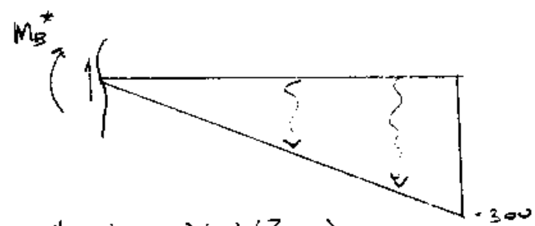
$\Sigma M_{A^*} = 0 = B_y^*(10) - \frac{1}{2}(300)(10)(10 + \frac{2}{3}(10))$

$0 = 10B_y^* - 25000$

$B_y^* = 2500\,k$ ✓

$\Sigma F_y = 0 = -300(10)(1/2) + A_y^* + 2500$

$A_y^* = -1000\,k$ or $1000^k \downarrow$ ✓



$0 = M_B^* + \frac{1}{2}(300)(10)(\frac{2}{3} \times 10)$

$M_B^* = -10{,}000$ ✓

$Y_B = \frac{M^*_B}{EI}$

$Y_B = \frac{-10{,}000 \text{ k}\cdot\text{ft}^3}{(4000\text{ksi})(3000\text{in}^4)} = -0.0008333 \times \cancel{144}\,(12)^3$

negative, so ∴ ↓

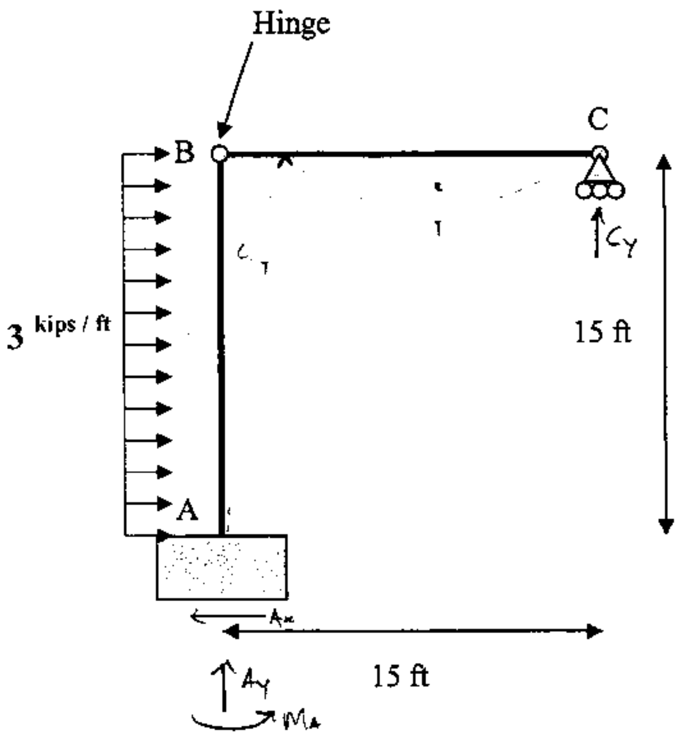
$\frac{k\ ft^2}{k/in^2\ in^4} = \frac{ft^2}{in^2} \times \frac{144\,in^2}{1\,ft^2}$

-1

$\boxed{Y_B = 0.12 \text{ in} \downarrow}$ (x12)

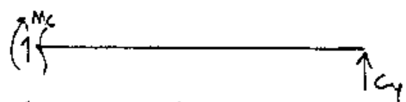
30.5

5. [40 pts] Solve for the <u>horizontal</u> deflection at point C using either the virtual work method or Castigliano's second theorem. Be sure to clearly note which method of analysis you have chosen. Include shear and moment diagrams for the real force system shown below as part of your solution. Let E = 29,000 ksi and I = 2,000 in^4. EI is constant for the entire frame.



$C_y = 0$

using equation of condition



$\Sigma M_C = 0 = C_y(15)$

$C_y = 0$ ✓

$\Sigma F_x = 0 = 3(15) - A_x$

$A_x = 45^k$ ✓

Analyzing the entire beam:

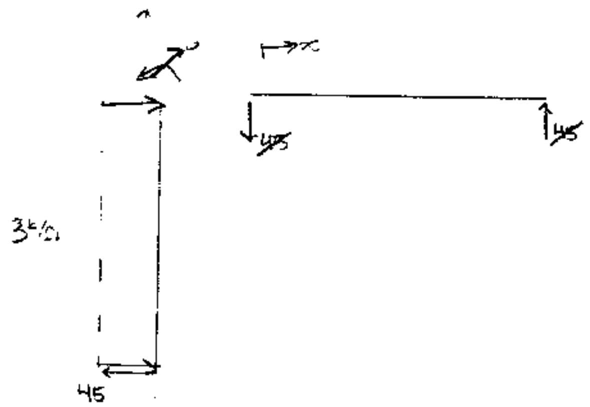
$\Sigma M_A = M_A - 3(15)(7.5)$

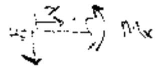
$M_A = 337.5$ ✓

$\Sigma F_y = 0 = C_y + A_y$

$0 = 0 + A_y$

$A_y = 0$ ✓

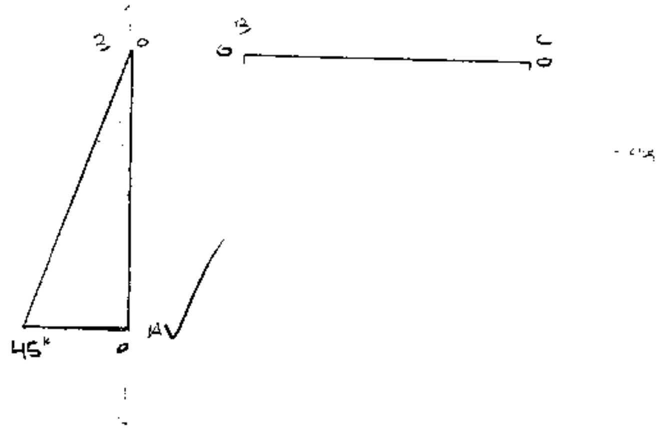




$\Sigma M(x) = 0 = M_x - \cancel{45(x)}$

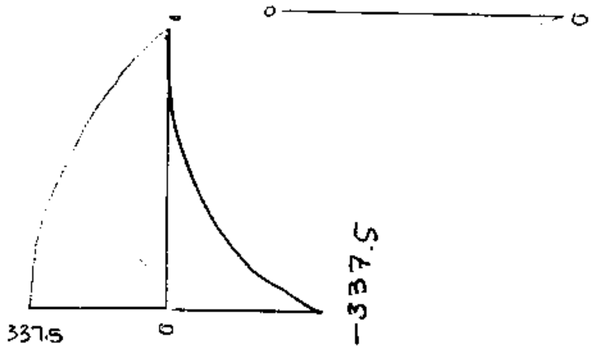
$M_x = \cancel{45(x)} = 0$

v(x) [kips]



$V_{AB}(x) = 45 - 3x$

m(x) [~~ft~~·kips]



$M_{AB}(x) = \int_0^{15} 45 - 3x \; dx$

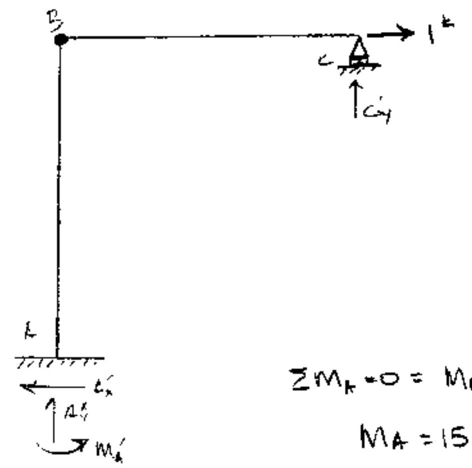
$M_{AB}(x) = 45x - \frac{3}{2}x^2 + C_1$

we know $M(x=0) = -337.5$ $\therefore$ $C_1 = -337.5$

$M_{AB}(x) = 45x - \frac{3}{2}x^2 - 337.5$ ✓

-7.5

Virtual Force System



$\Sigma M_B = 0 = C_y(15)$

$C_y = 0$

$\Sigma M_A = 0 = M_A + C_y(15) - 1(15)$

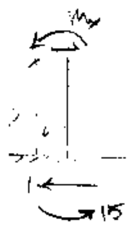
$M_A = 15$

$\Sigma F_x = 0 = 1 - A_x$

$A_x = 1$

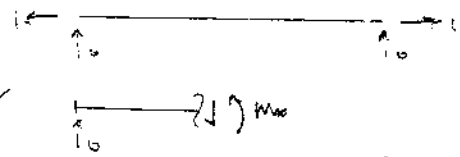
$\Sigma F_y = 0 = C_y + A_y$

$A_y = 0$



$\Sigma M_x = 0 = M_x + 15 - 1(x)$

$M_x = x - 15$ ✓



$\Sigma M_x = 0 = M_x$

$V(x) = 1$

$M(x) = \int 1\,dx$

$M(x) = x + C_2$

$M(0) = 0 = 15 + C_2$

$C_2 = -15$

Segment	Origin	Limits	M	m_v
AB	A	$0 \to 15$	$45x - \frac{3}{2}x^2 - 337.5$ ✓	$x - 15$ ✓
BC	B	$0 \to 15$	0	0

$$\Delta = \Sigma \int_0^L \frac{m_v M}{EI}\,dx$$

$$\Delta_c = \int_0^{15} \frac{45x - \frac{3}{2}x^2 - 337.5\,(x-15)}{EI}\,dx + \int_0^{15} \frac{(0)(0)}{EI}\,dx$$

$$\Delta_c = \int_0^{15} \frac{45x^2 - \frac{3}{2}x^3 - 337.5x - 675x + 22.5x^2 + 5062.5}{EI}\,dx + 0$$

(2)

$$\Delta_c = \int_0^{15} \frac{67.5x^2 - \frac{3}{2}x^3 - 1012.5x + 5062.5}{EI}\,dx$$

$$\therefore \Delta_c = \frac{1}{EI}\left(22.5x^3 - \frac{3}{8}x^4 - 506.25x^2 + 5062.5x\right|_0^{15} = \frac{18984.375}{EI}(12)^3 = .0003273 \;(144)$$

$= 0.04713$ in ; "+" ∴ →

$\boxed{\Delta_c = 0.0471 \text{ in} \rightarrow}$ x

5 Bonus (5 pts): Derive the following equation, which describes the amount of internal strain energy that develops in an axially loaded member when the member is gradually loaded with a force, in three steps or less.

$$\boxed{U_i = \frac{N^2L}{2AE}}$$

$U_e = \frac{1}{2}P\Delta = U_i$

where $P = N$

$\Delta = \frac{PL}{AE}$

$\therefore \quad U_i = \frac{1}{2}P\Delta = \frac{1}{2}(P)\left(\frac{PL}{AE}\right) = \frac{P^2L}{2AE}$ ✓

or

$U_i = \frac{N^2L}{2AE}$