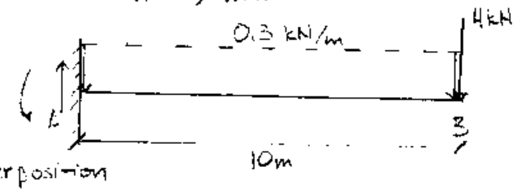
Assignment #6 | CE 461 | 1/11

10/11/06

49.5/50 + 20/20

8.28) Use the conjugate-beam method & determine the slope & deflection at B. The beam has a rectangular cross-section, where $E = 13$ GPa, $A = 0.08\ m^2$, $I = 0.500(10^{-3})\ m^4$.



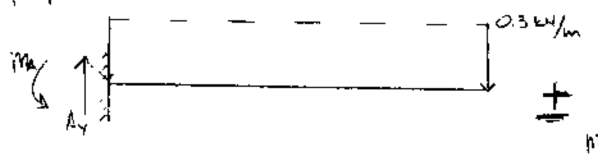
$\Sigma M_A = 0 = M_A + 0.3(10)(5) + 4(10)$

$M_A = -55$ kN·m

$\Sigma F_y = 0 = -.3(10) - 4 + A_y$

$A_y = 7$ kN

*using superposition



±

$\Sigma M_A = 0 = M_A - 0.3(10)(5)$

$M_A = 15$ kN·m

$\Sigma F_y = 0 = A_y - 10(.3)$

$A_y = 3$ kN

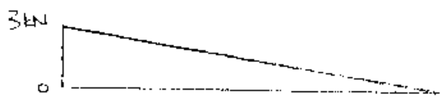
$\Sigma M_A' = 0 = M_A' - 4(10)$

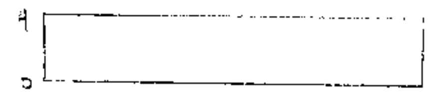
$M_A' = 40$ kN

$\Sigma F_y = 0 = A_y - 4$

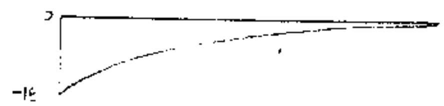
$A_y = 4$ kN

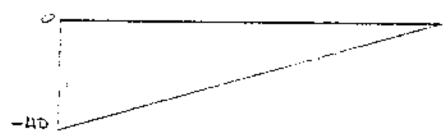
V(x)

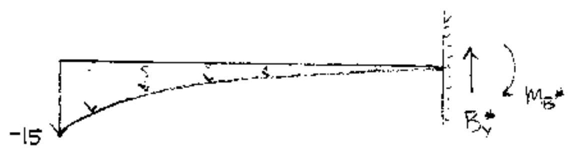


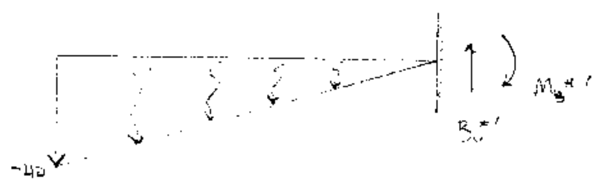


M(x)









$\Sigma M_B^* = 0 = M_B^* - \frac{1}{3}(15)(10)\left[\frac{3}{4}(10)\right]$

$M_B^* = -375$ kN m

$\Sigma F_y = 0 = -15(10)(\frac{1}{3}) + B_y^*$

$B_y^* = 50$ kN

$\Sigma M_{B^{*\prime}} = 0 = M_B^{*\prime} - \frac{1}{2}(40)(10)(10 - \frac{10}{3})$

$M_B^{*\prime} = -1333.33$ kN·m

$\Sigma F_y = 0 = -\frac{1}{2}(40)(10) + B_y^*$

$B_y^* = 200$ kN

$\theta_B = B_y^* = \frac{50 \text{ kN}}{EI} = \frac{50}{(13,000,000)(.5\times10^{-3})}$

$\theta_B = .00769$ rad

$\theta_B = B_y^{*\prime} = \frac{200}{(13,000,000)(.5\times10^{-3})}$

$\theta_B' = 0.0308$ rad

$Y_B = M_B^* = \frac{-375}{EI} = \frac{-375}{(13,000,000)(.5\times10^{-3})}$

$Y_B = 0.0577$ m

$Y_B' = M_B^{*\prime} = \frac{-1333.33}{(13,000,000)(.5\times10^{-3})}$

$Y_B' = 0.205$ m

$\theta_B = 0.00769 + 0.0308 = 0.03849$ rad

$\boxed{\theta_B = 0.0385 \text{ rad}}$

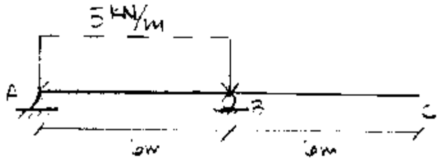
$Y_B = -.0577 + -.205 = -.2627$ m

$\boxed{Y_B = -0.263 \text{ m}}$

10/11/06

9.5

7.32) Use the conjugate-beam method & determine the deflection at C. Assume A is a pin & B is a roller.



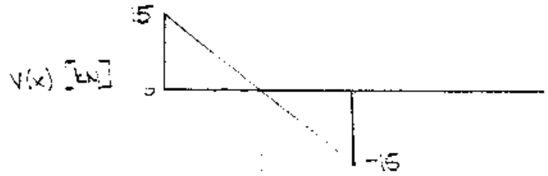
$\Sigma M_A = 0 = 5(6)(3) - B_y(6)$

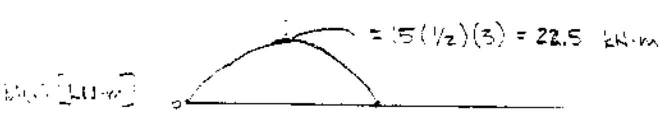
$B_y = 15$ kN

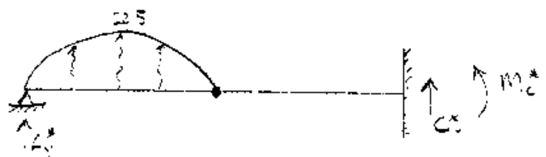
$\Sigma F_y = 0 = -5(6) + 15 + A_y$

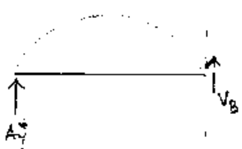
$A_y = 15$ kN

OK









Using equation of condition provided by internal hinge

$\Sigma M_B = 0 = A_y^*(6) - A_{parabola}$

$0 = A_y^*(6) - \left[\frac{2}{3}(22.5)(3)\right] \times 2\ (3)$

$A_y^* = 45$ kN

Using the entire beam

$\Sigma F_y = 0 = 45 - \left(\frac{2}{3}(22.5)(6)\right) + C_y^*$

$C_y^* = 45$

$\Sigma M_c^* = 0 = M_c^* - \frac{2}{3}(22.5)(6)(9) - 45(12)$

-1/2 (sign)

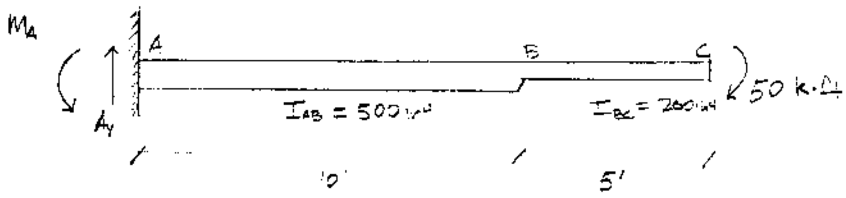
$M_c^* = \cancel{1350}$

$Y_c = M_c^* = \cancel{\frac{1350}{EI}}$

$\boxed{Y_c = \cancel{\frac{1350}{EI}}\ (m)}$

01/11/06

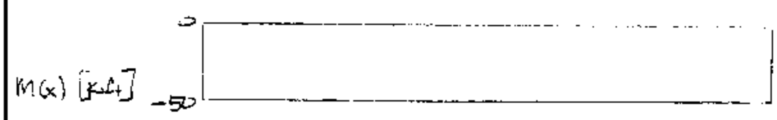
8.33) Use the conjugate-beam method & determine the deflection & slope at end C of the cantilever beam. $E = 29 \times 10^3$ ksi. The moment of inertia of each segment is indicated in the figure.

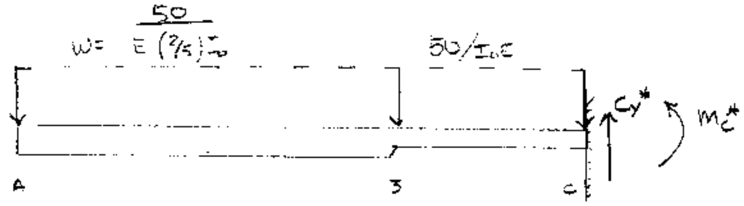


$\Sigma M_A = 0 = -M_A - 50$

$M_A = -50$

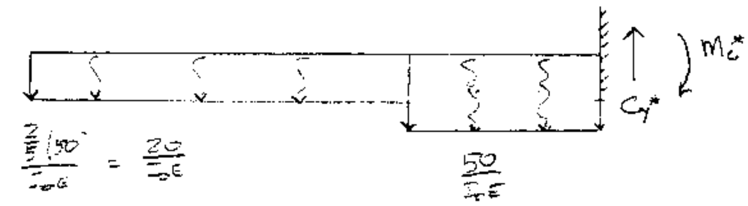
$\Sigma F_y = 0 = A_y$





Let $I_o = I_{BC} = 200 \text{ in}^4$

$I_o = I_{BC} = \frac{2}{5} I_{AB}$



$$\Sigma M_C = 0 = M_C^* - \left(\frac{50}{I_oE}\right)(5)(2.5) - \left(\frac{20}{I_oE}\right)(10)(10)$$

$$M_C^* = \frac{2625}{I_oE}$$

$$\Sigma F_y = 0 = -\left(\frac{20}{I_oE}\right)(10') - \left(\frac{50}{I_oE}\right)(5') + C_y^*$$

$$C_y^* = \frac{450}{EI_o}$$

$$\theta_C = C_y^* = \frac{450}{(29{,}000)(200)(1/144)} = 0.0112 \text{ rad}$$

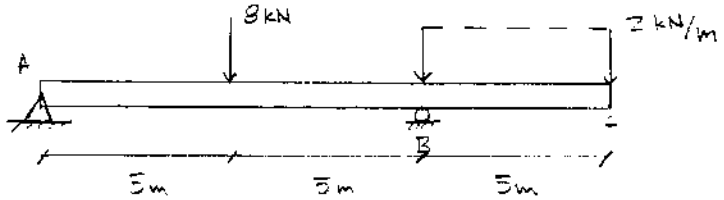
$$\boxed{\theta_C = 0.0112 \text{ rad}}$$

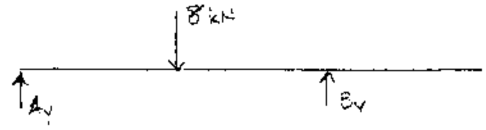
$$Y_C = M_C^* = \frac{-2625}{(29{,}000)(200)(1/144)} = -0.0652 \text{ ft} \times 12 \text{ in/ft} = -0.782$$

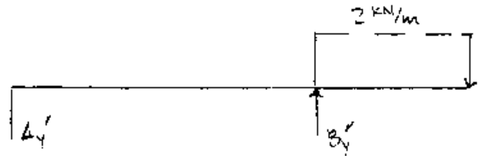
$$\boxed{Y_C = 0.782 \text{ in} \downarrow}$$

10/11/06

8.36) Determine the slope at the end C of the beam. Use the conjugate-beam method. $E = 200$ GPa, $I = 70(10^6)$ mm^4. Assume B is a roller & is a pin.







$\Sigma M_A = 0 = -8(5) + B(10)$

$B_y = 4$ kN

$\Sigma F_y = 0 = -8 + 4 + A_y$

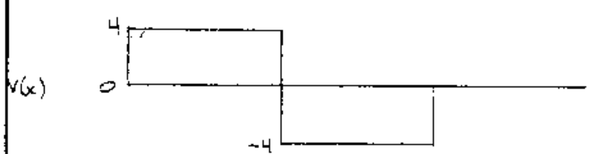
$A_y = 4$ kN

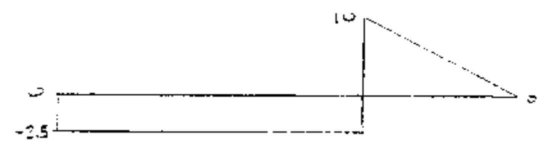
$\Sigma M_A = 0 = B_y'(10) - 2(5)(12.5)$

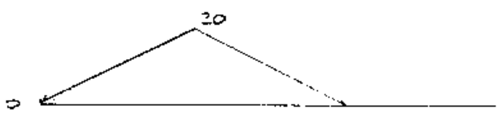
$B_y' = 12.5$ kN

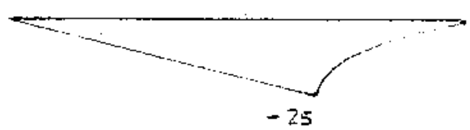
$\Sigma F_y = 0 = -2(5) + 12.5 + A_y'$

$A_y' = -2.5$ kN

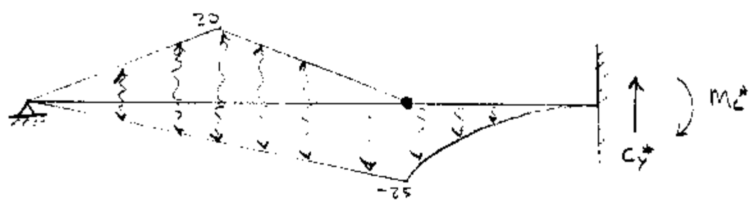








recombine loads to form conjugate beam



use equation of condition to solve for A_y^*

$\Sigma M_B = 0 = A_y(10) + \frac{1}{2}(20)(5)(5 + \frac{1}{3}5) + \frac{1}{2}(20)(5)(5 \cdot \frac{2}{3}) - \frac{1}{2}(25)(10)(\frac{10}{3})$

$A_y = -8.3$

Sum forces in y direction on whole beam to solve for C_y^*

$\Sigma F_y = 0 = \frac{1}{2}(20)(5) + \frac{1}{2}(20)(5) - \frac{1}{2}(25)(10) - \frac{1}{3}(25)(5) - A^* + C_y^*$

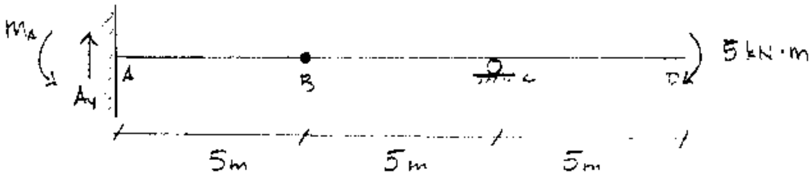
$C_y^* = \frac{75}{EI} = \frac{75}{(200 \times 10^6)(7 \times 10^{-5})} = 0.005357$

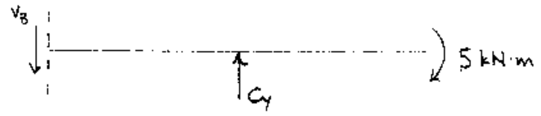
$\theta_c = C_y^* = 0.00536$ rad

$\boxed{\theta_c = 0.00536 \text{ rad}}$

10/11/06

8.37) Use the conjugate-beam method & determine the slope just to the left & just to the right of the pin at B. Also determine the deflection at D. Assume the beam is fixed support at A, & C is a roller. EI is constant





$\Sigma M_B = 0 = 5 - C_y(5)$

$C_y = 1$ kN

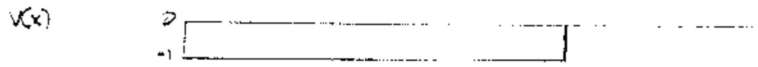
Analyzing the whole beam

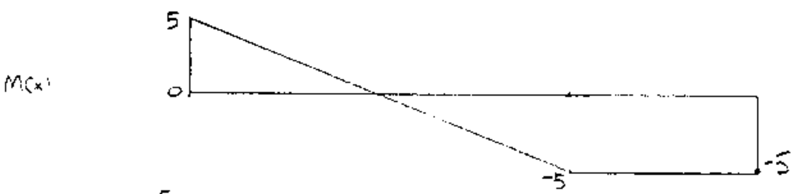
$\Sigma M_A = 0 = M_A - 5 + 1(10)$

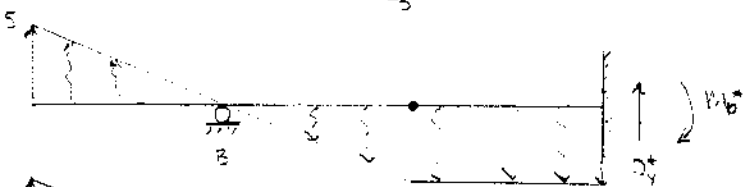
$M_A = -5$ kN·m

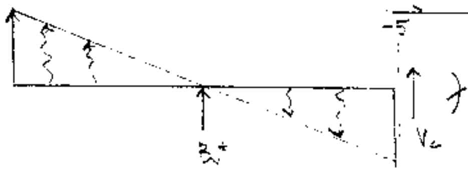
$\Sigma F_y = 0 = A_y + 1$

$A_y = -1$ kN









using equation of condition

$\Sigma M_c = 0 = \frac{1}{2}(5)(5)(5 + \frac{2}{3}5) - \frac{1}{2}(5)(5)(\frac{5}{3}) + B_y^*(5)$

$B_y^* = -16.67$ kN

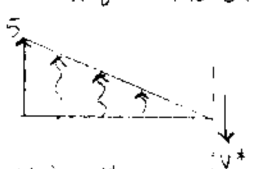
analyzing the entire beam:

$\Sigma F_y = 0 = \frac{1}{2}(5)(5) + (-16.67) - \frac{1}{2}(5)(5) - 5(5) + D_y^*$

$D_y^* = 41.67$ kN

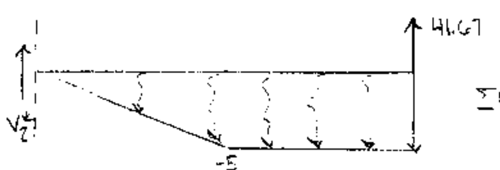
$\Sigma M_D = 0 = M_D - (5)(5)(2.5) - \frac{1}{2}(5)(5)(5 + \frac{5}{3}) + (-16.67)(10) + \frac{1}{2}(5)(5)(10 + \frac{2}{3}\cdot 5)$

$M_D^* = 145.87$ kN·m



$\Sigma F_y = 0 = \frac{1}{2}(5)(5) - V_1^*$

$V_1^* = 12.5$



$\Sigma F_y = 0 = 41.67 - (5)(5) - \frac{1}{2}(5)(5) + V_2^*$

$V_2^* = 4.17$

$\boxed{\theta_{LB} = \frac{12.5}{EI} \text{ rad}}$

$\boxed{\theta_{RB} = \frac{4.17}{EI} \text{ rad}}$

8.37 Continued...

Deflection at D...

$$Y_D = M_D^* = \frac{-145.87}{EI}$$

$$\boxed{Y_D = \frac{-145.87}{EI}}$$

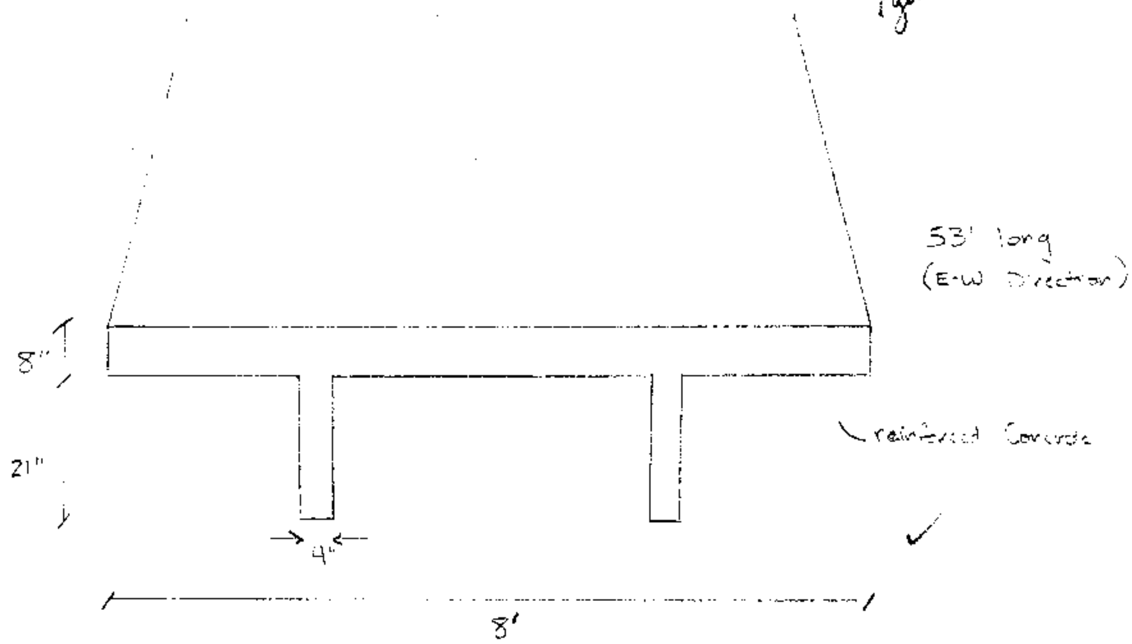
10/11/06

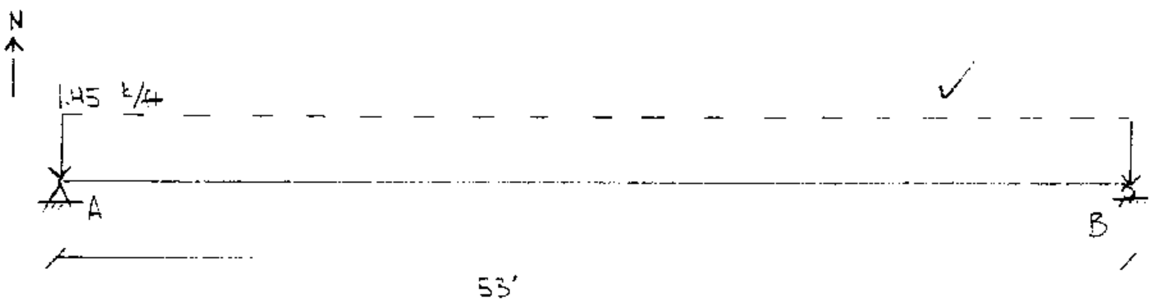
Problem #6

Estimate the midspan deflection of one of the loaded, interior double-tee beams running in the E-W direction on the parking level circled in the parking garage North of Allen Fieldhouse.
Use conjugate beam method.

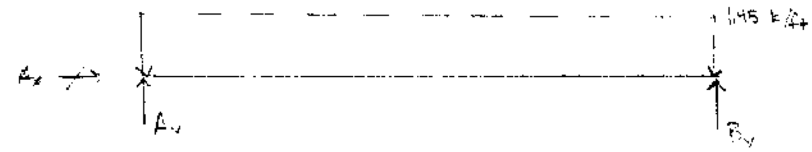
Assumptions: ✓

* Use information from assignment 1, including dimensions, loading, tributary area.
* Use dimensions acquired on site. ✓
* Assume continuous loading over the deck of the beam. ✓
* Assume constant average width for webs of 4" ✓
* Load from C. Bennett's Solutions $P = 1.45\ ^{kip}/_{ft}$ (off course website) ✓
* Assume A is a pin & B is a roller on the free body diagram below. ✓
* Assume a value of E = 4000 ksi = 4,000,000 psi – C. Bennett 10/11/06 ✓
* Assume the connection or joint to adjacent double-tee beams provide no or negligible support to the beam being analyzed. ✓ (good)





Problem 1 Continued...

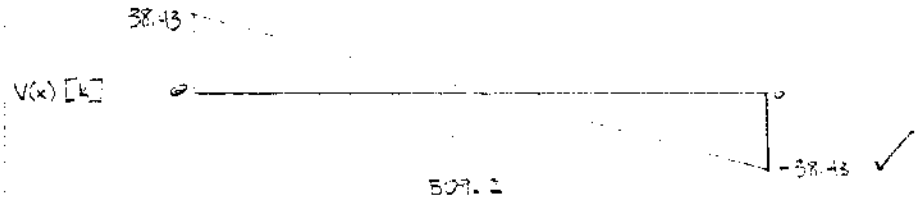


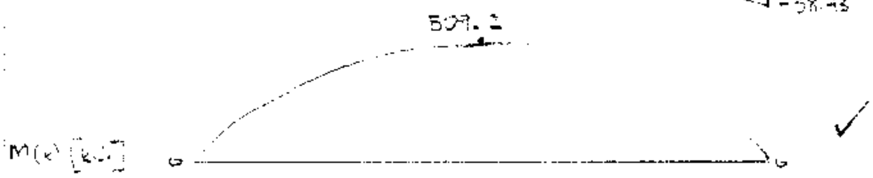
$\Sigma M_A = 0 = 1.45(53)(53/2) - B_y(53)$

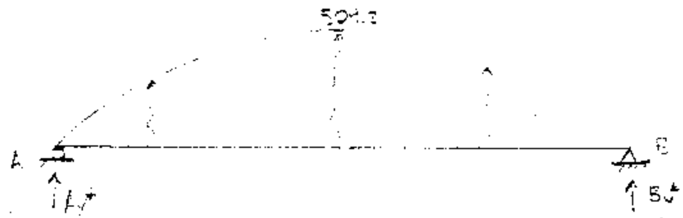
$B_y = 38.43$ k

$\Sigma F_y = 0 = -1.45(53) + 38.43 + A_y$

$A_y = 38.43$ k







Conjugate Beam w/ Load

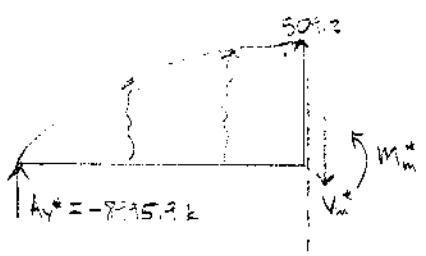
$\Sigma M_{B^*} = 0 = A_y^*(53) + \left(2/3(53)(509.2)(53/2)\right)$

$A_y^* = -8995.9$ k ✓

$\Sigma F_y = 0 = 2/3(53)(509.2) + (-8995.9) + B_y^*$

$B_y^* = -8995.9$ k ✓

- Make a cut at the midspan to solve for the internal moment in the conjugate beam, which will be the midspan deflection for the real beam.



$\Sigma M_m = 0 = M_m^* - 2/3(509.2)(53/2)\left(\frac{3}{8}\cdot 53/2\right) - (-8995.9)(53/2)$

$M_m^* = -148994.9$

$Y_{midspan} = Y_m = \dfrac{-148994.9}{EI}$ ✓

10/11/06

Problem 6 Continued...

Calculate I for beam

$$\bar{y}_c = \frac{(4/12)(21/12)(21/12 \cdot \frac{1}{2}) + (4/12)(21/12)(21/12 \cdot 1/2) + (8/12)(8)(21/12 + 4/12)}{(4/12)(21/12) + (4/12)(21/12) + (8/12)(8)}$$

$$\bar{y}_c = \frac{12.13}{6.5} = 1.866 \text{ ft}$$ ✓

$$I = 2\left[1/12\,(4/12)(21/12)^3 + (4/12)(21/12)\,(1.866 - 21/12)\right] + 1/12\,(8)(8/12)^3 + (8)(8/12)\left((21/12 + 4/12) - 1.866\right)$$

$$I = 2(0.2165) + 1.3566$$

$$I = 1.7896 \text{ ft}^4 = 37109.15 \text{ in}^4$$ ✓

$$Y_m = \frac{-148994.9}{EI} = \frac{-148994.9}{(4{,}000 \text{ ksi})\,(37109.15 \text{ in}^4)(1/144)} = -0.1445 \text{ ft}$$

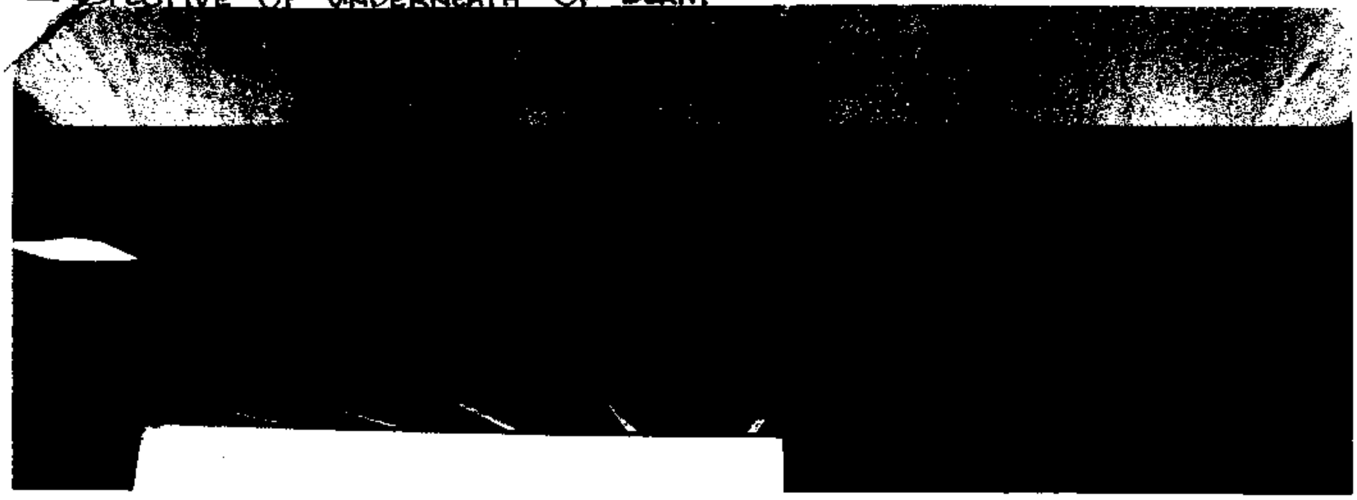
$$Y_m = -0.1445 \text{ ft} \times 12 \text{ in/ft} = -1.73 \text{ in}$$ ✓

> The deflection at the mid span of one of the double-tee beams is equal to -1.73 in or 1.73 in. down. ✓

Brandon - This is excellent work. Your assumptions are valid, your methodology sound, & presentation more than sufficient. Thank you for your efforts.

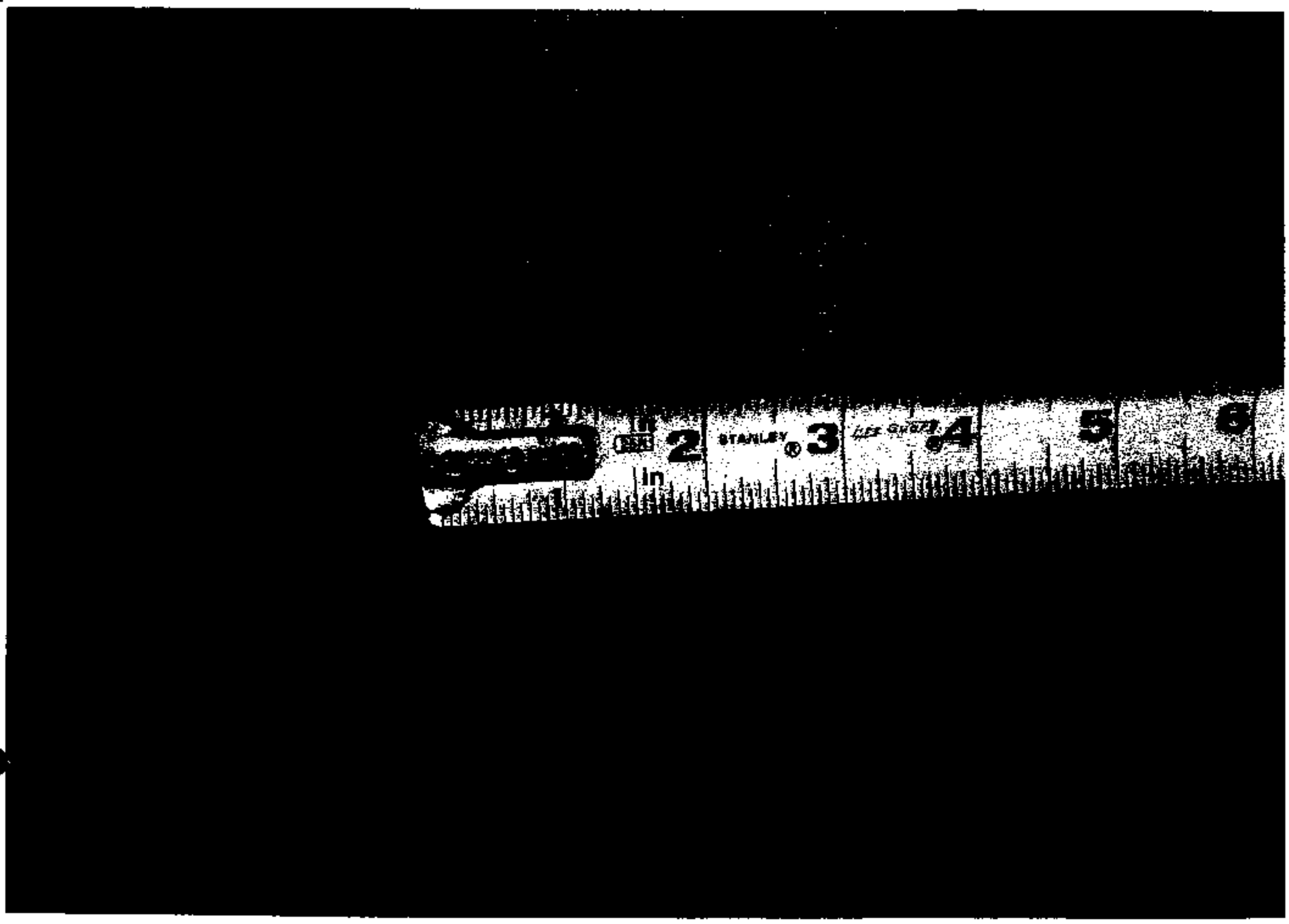
20/20

SPECTIVE OF UNDERNEATH OF BEAM



THICKNESS OF SLAB

DEPTH OF WEB



THICKNESS OF WEB