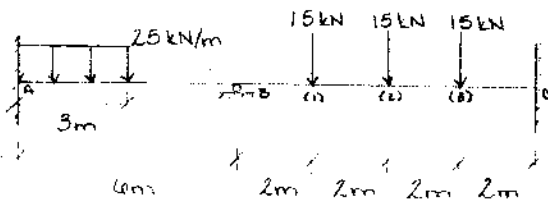


(Problem 12-1) Determine the moments at the supports. EI is constant. Assume B is a roller and A and C are fixed.



$$\frac{48.5}{30} = 3$$

FIXED END MOMENTS

$$FEM_{AB} = \frac{wL^2}{12L^2} (6L^2 - 8aL + 3a^2) = \frac{25(3)^2}{12(6)^2} (6(6)^2 - 8(3)(6) + 3(3)^2) = 51.5625 \text{ kN}\cdot\text{m} \uparrow$$

$$FEM_{BA} = -\frac{wL^2}{12L^2} (4L - 3a) = -\frac{25(3)^2}{12(6)^2} (4(6) - 3(3)) = -23.4375 \text{ kN}\cdot\text{m} \downarrow$$

$$FEM_{BC1} = \frac{Pab^2}{L^2} = \frac{15(2)(4)^2}{8^2} = 16.875 \text{ kN}\cdot\text{m} \uparrow$$

$$FEM_{BC2} = \frac{PL}{8} = \frac{15(8)}{8} = 15 \text{ kN}\cdot\text{m} \uparrow$$

$$FEM_{BC3} = \frac{Pab^2}{L^2} = \frac{15(4)(2)^2}{8^2} = 5.625 \text{ kN}\cdot\text{m} \uparrow$$

$$FEM_{BC} = 37.5 \text{ kN}\cdot\text{m} \uparrow$$

$$FEM_{CB1} = -\frac{Pb^2a}{L^2} = -\frac{15(2)^2(4)}{8^2} = -5.625 \text{ kN}\cdot\text{m} \downarrow$$

$$FEM_{CB2} = -\frac{PL}{8} = -\frac{15(8)}{8} = -15 \text{ kN}\cdot\text{m} \downarrow$$

$$FEM_{CB3} = -\frac{Pb^2a}{L^2} = -\frac{15(4)(2)^2}{8^2} = -10.875 \text{ kN}\cdot\text{m} \downarrow$$

$$FEM_{CB} = -37.5 \text{ kN}\cdot\text{m} \downarrow$$

DISTRIBUTION FACTORS

$$DF_{BA} = \frac{I_B}{I_A + I_B} = 0.571$$

$$DF_{BC} = \frac{I_B}{I_A + I_B} = 0.429$$

TABLE

MEMBER	AB	BA	BC	CB
DIST. FACTORS		0.571	0.429	
FEMs	51.5625	-23.4375	37.5	-37.5
BALANCE B		8.0297	-6.0328	
CARRYOVER	-4.0149			3.0164
FINAL	47.5476	-31.4072	31.4672	-40.5164

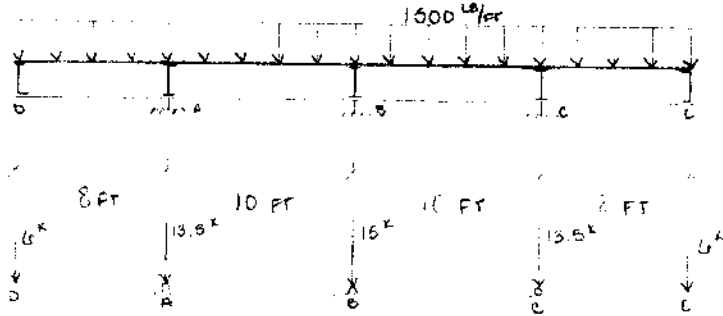
$$M_A = M_{AB} = 47.5476 \text{ kN}\cdot\text{m} \leftarrow \uparrow$$

$$M_C = M_{CB} = -40.5164 \text{ kN}\cdot\text{m} \leftarrow \downarrow$$

M_A
 M_C

(Problem 12-6) Determine the moments at A, B, and C and then draw the moment diagram for the girder DE. EI is constant. Assume the support at B is a pin and A and C are rollers. The distributed load rests on simply supported floor boards that transmit the load to the floor beams.

10
10



FIXED END MOMENTS

$$\begin{aligned}
 FEM_{DA} &= 0 & FEM_{BC} &= 0 \\
 FEM_{AD} &= 6(8) = -48 \text{ k}\cdot\text{ft} \downarrow & FEM_{CB} &= 0 \\
 FEM_{AB} &= 0 & FEM_{CE} &= 6(8) = 48 \text{ k}\cdot\text{ft} \uparrow \\
 FEM_{BA} &= 0 & FEM_{EC} &= 0
 \end{aligned}$$

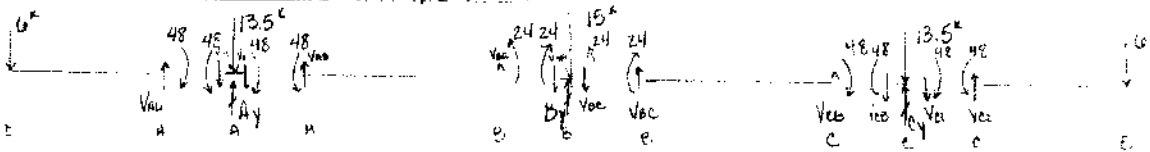
DISTRIBUTION FACTORS

$$\begin{aligned}
 DF_{AD} &= 0 & DF_{BA} &= 0.5 & DF_{CB} &= 1 \\
 DF_{AB} &= 1 & DF_{BC} &= 0.5 & DF_{CE} &= 0
 \end{aligned}$$

TABLE

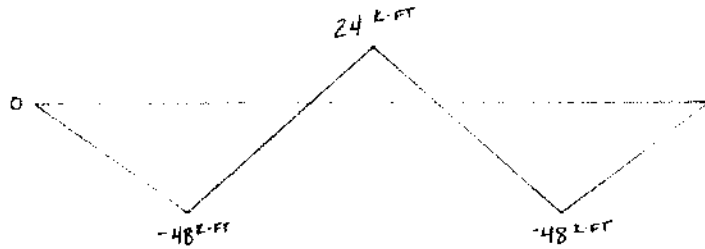
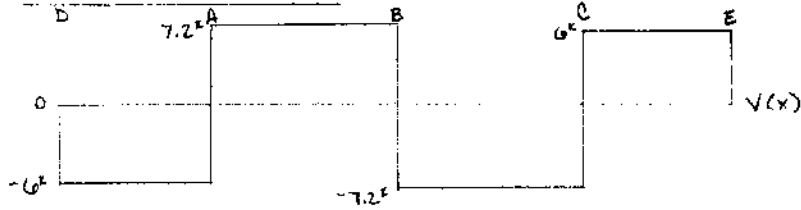
MEMBER	DA	AD	AB	BA	BC	CB	CE	EC
DIST FACTORS		0	1	0.5	0.5	1	0	
FEM'S	0	-48	0	0	0	0	+48	0
BALANCE			+48			-48		
CARRYOVER				+24	-24			
FINAL	0	-48	+48	+24	-24	-48	+48	0

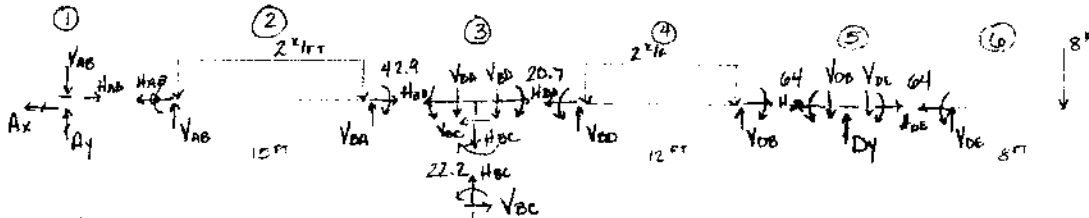
$$\begin{aligned}
 M_{AD} &= -48 \text{ k}\cdot\text{ft}, (-) \downarrow & M_{AB} &= 48 \text{ k}\cdot\text{ft}, (+) \uparrow \\
 M_{BA} &= 24 \text{ k}\cdot\text{ft}, (+) \uparrow & M_{BC} &= -24 \text{ k}\cdot\text{ft}, (-) \downarrow \\
 M_{CB} &= -48 \text{ k}\cdot\text{ft}, (-) \downarrow & M_{CE} &= 48 \text{ k}\cdot\text{ft}, (+) \uparrow
 \end{aligned}$$



$$\begin{aligned}
 \sum F_{D-A} &= 0 \Rightarrow V_{AD} = 6^k \\
 \sum F_{C-E} &= 0 \Rightarrow V_{CE} = 6^k \\
 \sum M_B = 0 & \Rightarrow 48 + V_{AB}(10) - 24 = 0 \Rightarrow V_{AB} = 7.2^k \\
 \sum M_A = 0 & \Rightarrow 48 - 24 + V_{BA}(10) = 0 \Rightarrow V_{BA} = -7.2^k \\
 \sum M_C = 0 & \Rightarrow 24 + 48 + V_{BC}(10) = 0 \Rightarrow V_{BC} = -7.2^k \\
 \sum M_B = 0 & \Rightarrow 24 + 48 - V_{CB}(10) = 0 \Rightarrow V_{CB} = 7.2^k \\
 \sum F_{A-B} &= 0 \Rightarrow A_y = 26.7^k \uparrow \\
 \sum F_{B-C} &= 0 \Rightarrow B_y = 0.6^k \uparrow \\
 \sum F_{C-E} &= 0 \Rightarrow C_y = 26.7^k \uparrow
 \end{aligned}$$

SHEAR AND MOMENT DIAGRAMS



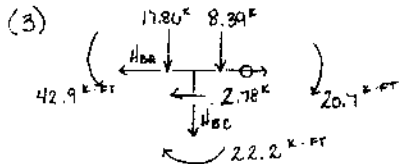


(7) $\sum M_{bc} = 0$
 $22.2 + 11.1 - V_{cb}(12) = 0$
 $\therefore V_{cb} = 2.78 \text{ k}$
 $\rightarrow \sum F_x = 0$
 $\therefore V_{bc} = 2.78 \text{ k}$

(2) $\sum M_B = 0 + 42.9 - V_{AB}(15) - 2(15)(7.5)$
 $\therefore V_{AB} = 12.14 \text{ k}$
 $\uparrow \sum F_y = 0 \quad V_{AB} + V_{BA} - 2(15) = 0$
 $\therefore V_{BA} = 17.86 \text{ k}$

(6) $\uparrow \sum F_y = 0 \quad \therefore V_{DE} = 8 \text{ k}$
 (4) $\sum M_D = 0 = -20.7 + 64 - V_{DB}(12) + 2(12)(6)$
 $\therefore V_{DB} = 15.61 \text{ k}$
 $\uparrow \sum F_y = 0 = V_{DB} + V_{BD} - 2(12)$
 $\therefore V_{BD} = 8.39 \text{ k}$

(6) $H_{DE} = 0 \rightarrow H_{BD} = 0$

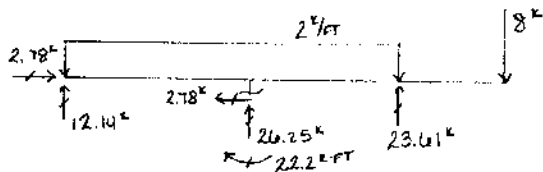


$H_{BA} = -17.86 \text{ k}$
 $H_{BC} = -20.7 \text{ k}$

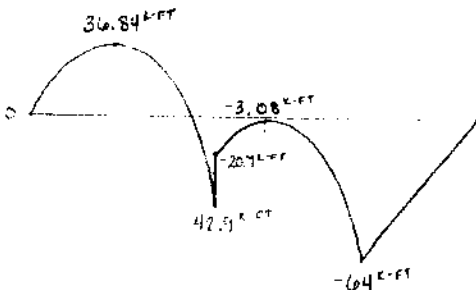
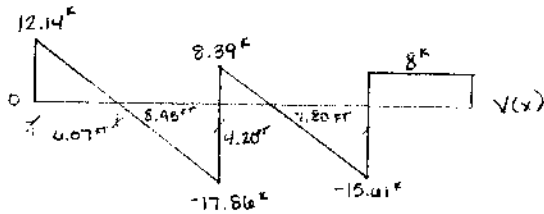
$\therefore H_{CB} = -26.25$

REACTIONS

$M_C = 11.1 \text{ k-ft} \uparrow$
 $C_y = 26.25 \text{ k} \uparrow$
 $C_x = 2.78 \text{ k} \leftarrow$
 $A_y = 12.14 \text{ k} \uparrow$
 $A_x = 2.78 \text{ k} \rightarrow$
 $D_y = 23.61 \text{ k} \uparrow$



SHEAR AND MOMENT DIAGRAMS



5/1/2006

1240831

CE-4661

ASSIGNMENT #13

6/9

(Problem 12-16) Determine the moments at the ends of each member of the frame. The members are fixed connected at the supports and joints. EI is the same for each member.

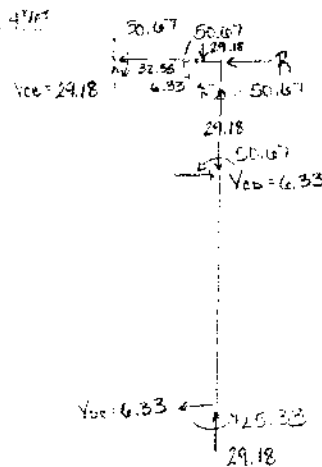
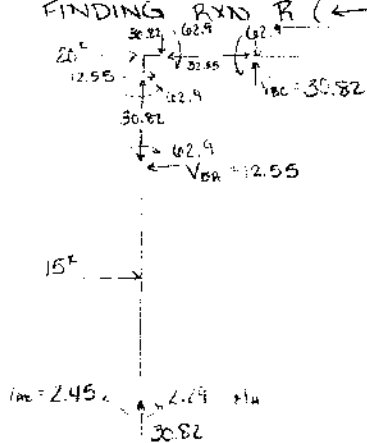
PART: WITHOUT SIDESWAY

$$\begin{aligned}
 FEM_{CD} = FEM_{DC} &= 0 \\
 FEM_{AB} &= \frac{PL}{8} = \frac{12(12)}{8} = +22.5 \text{ K-FT} \\
 FEM_{BA} &= -\frac{PL}{8} = -\frac{12(12)}{8} = -22.5 \text{ K-FT}
 \end{aligned}$$

$$\begin{aligned}
 FEM_{BC} &= \frac{wL^2}{12} = \frac{4(15)^2}{12} = +75 \text{ K-FT} \\
 FEM_{CB} &= -\frac{wL^2}{12} = -\frac{4(15)^2}{12} = -75 \text{ K-FT}
 \end{aligned}$$

MEMBER	AB	BA	BC	CB	CD	DC
DIST. FACT.		0.556	0.444	0.444	0.556	
FEMs	+22.5	-22.5	+75	-75	0	0
BALANCE		-29.19	-23.31	+33.3	+41.7	
CARRYOVER	-14.595		-11.655	-11.655		+20.85
BALANCE		-9.2574	-7.3926	+5.1748	+6.4802	
CARRYOVER	-4.6287		+2.5874	+3.6943		+3.2401
BALANCE		-1.4386	-1.1488	+1.0412	+2.0551	
CARRYOVER	-0.7193		+0.5206	+0.5744		+1.0276
BALANCE		-0.4563	-0.3643	+0.2650	+0.3144	
CARRYOVER	-0.2281		+0.1825	+0.1573		+0.1573
BALANCE		-0.0225	-0.0180	+0.0126	+0.0157	
FINAL	+2.2933	-2.9357	-2.9357	-50.6717	+50.6717	+25.3281

FINDING R AND R (←)



$$\begin{aligned}
 (+ \sum M_B = 0) &= -62.9 + 2.29 - V_{CB}(12) + 15(6) \\
 V_{AB} &= 2.45 \text{ K} \\
 (+ \sum M_D = 0) &= -62.9 - 4(15)(7.5) + V_{CB}(15) - 50.67 \\
 V_{CB} &= 29.18 \text{ K} \\
 (+ \sum M_D = 0) &= -50.67 - V_{CB}(12) + 6.33 \\
 V_{CB} &= 6.33 \text{ K} \\
 R &= 26.22 \text{ K}
 \end{aligned}$$

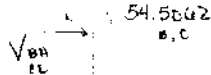
PART 2 : WITH SIDESWAY

$FEM_{bc} = FEM_{cb} = 0$

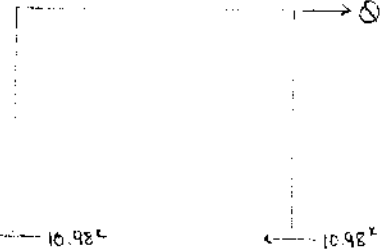
$FEM_{ab} = FEM_{ba} = FEM_{ed} = FEM_{de} = \frac{+0.01 \Delta}{L^2}$
 ASSUME $\frac{0.01 \Delta}{L^2} = 100 \text{ } 1-k$

MEMBER	AB	BA	BC	CB	CD	DC
DIST. FACT		0.556	0.444	0.444	0.556	
FEMs	+100	+100	0	0	+100	+100
BALANCE		-55.6	-44.4	-44.4	-55.6	
CARRYOVER	-27.8		-22.2	-22.2		-27.8
BALANCE		+12.3432	+9.8568	+9.8568	+12.3432	
CARRYOVER	+6.1716		+4.9284	+4.9284		+6.1716
BALANCE		-2.7402	-2.1882	-2.1882	-2.7402	
CARRYOVER	-1.3701		-1.0941	-1.0941		-1.3701
BALANCE		+0.6083	+0.4858	+0.4858	+0.6083	
CARRYOVER	+0.3042		+0.2429	+0.2429		+0.3042
BALANCE		-0.1361	-0.1078	-0.1078	-0.1361	
CARRYOVER	-0.0676		-0.0539	-0.0539		-0.0676
BALANCE		+0.0300	+0.0239	+0.0239	+0.0300	
FINAL	+77.2381	+54.5062	-54.5062	-54.5062	+54.5062	+77.2381

FINDING Q (→)



$\sum M_{AB} = 0 \quad 54.5062 + 77.2381 - V_{BA}(12) = 0$
 $V_{BA} = V_{CD} = 10.98 \text{ } k$
 $V_{AB} = V_{DC} = 10.98 \text{ } k$



$Q = 21.96$

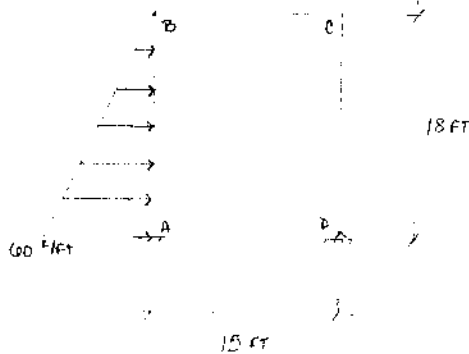
MEMBER END MOMENTS ($M = M_0 + (\frac{R}{8})M_0$)

$M_{AB} = 2.2933 + (\frac{21.96}{21.96})(77.2381) \quad \therefore M_{AB} = 95.51 \text{ } k\text{-ft, (+)}$
 $M_{BA} = -62.9357 + (\frac{21.96}{21.96})(54.5062) \quad \therefore M_{BA} = 2.14 \text{ } k\text{-ft, (+)}$
 $M_{BC} = 62.9357 + (\frac{21.96}{21.96})(-54.5062) \quad \therefore M_{BC} = -2.14 \text{ } k\text{-ft, (-)}$
 $M_{CB} = 50.6717 + (\frac{21.96}{21.96})(-54.5062) \quad \therefore M_{CB} = -115.75 \text{ } k\text{-ft, (-)}$
 $M_{CD} = 50.6717 + (\frac{21.96}{21.96})(54.5062) \quad \therefore M_{CD} = 115.75 \text{ } k\text{-ft, (+)}$
 $M_{DC} = 25.3281 + (\frac{21.96}{21.96})(77.2381) \quad \therefore M_{DC} = 117.55 \text{ } k\text{-ft, (+)}$

← END MOMENTS

(Problem 12-19) The side of the frame is subjected to the hydrostatic loading shown. Determine the moments at each joint and support. EI is constant.

8.5
10



DISTRIBUTION FACTORS

$$DF_{BA} = \frac{I_{AB}}{I_{AB} + I_{BC}} = 0.4545$$

$$DF_{BC} = \frac{I_{BC}}{I_{AB} + I_{BC}} = 0.5455$$

$$DF_{CB} = \frac{I_{BC}}{I_{BC} + 3I_{CD}} = 0.6154$$

$$DF_{CD} = \frac{3I_{CD}}{I_{BC} + 3I_{CD}} = 0.3846$$

PART I: WITHOUT SIDESWAY

$$FEM_{AB} = \frac{+wL^2}{20} = \frac{40(15)^2}{20} = +972 \text{ k}\cdot\text{ft}$$

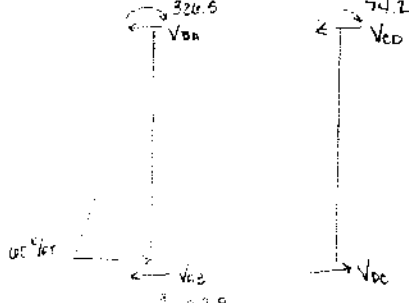
$$FEM_{BA} = \frac{-wL^2}{30} = \frac{-40(15)^2}{30} = -648 \text{ k}\cdot\text{ft}$$

$$FEM_{BC} = FEM_{CB} = FEM_{CD} = FEM_{DC} = 0$$

MEMBER	AB	BA	BC	CB	CD	DC
DIST. FACT.		0.4545	0.5455	0.6154	0.3846	
FEMS	+972	-648	0	0	0	0
BALANCE		294.516	353.484			
CARRYOVER	147.258			176.742		
BALANCE				108.767	67.975	
CARRYOVER			-54.384			0
BALANCE		74.718	29.066			
CARRYOVER	42.389			14.833		
BALANCE				-9.128	-5.705	
CARRYOVER			-4.564			0
BALANCE		2.074	2.490			
CARRYOVER	1.037			1.245		
BALANCE				-0.766	-0.479	
CARRYOVER			-0.383			0
BALANCE		0.174	0.209			
CARRYOVER	0.087			0.105		
BALANCE				-0.065	-0.040	
CARRYOVER			-0.033			0
BALANCE		0.015	0.018			
FINAL	+1132.771	+326.503	326.503	-74.199	+74.199	0

OK

FINDING REACTION R (←)



$$\sum M_A = 0 = 326.5 + V_B(15) + V_C(18) + 1132.8$$

$$V_B = 135.21 \text{ k}$$

$$\sum M_B = 0 = -74.2 + V_C(18) = 0$$

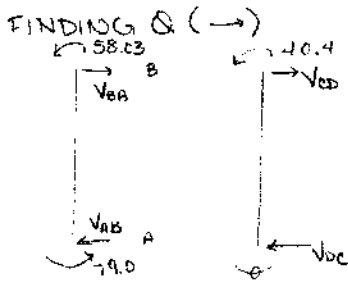
$$V_C = 4.12 \text{ k}$$

$$R = 139.33 \text{ k}$$

1/2 sign

PART 2: WITH SIDESWAY

UNIFORM	→ 1.145	→ 1.404	→ 2.941	→ 2.495	← 1.145
CARRYOVER	+0.587	-1.996	+0.705	-0.705	0
BALANCE	-0.907	-1.089	+0.434	-0.271	0
CARRYOVER	-0.454	-0.217	+0.545	-0.545	0
BALANCE	+0.099	+0.118	+0.935	+0.210	0
CARRYOVER	+0.050	+0.148	+0.059	-0.059	0
BALANCE	-0.076	-0.092	-0.036	-0.023	0
CARRYOVER	-0.038	-0.018	-0.046	+0.046	0
BALANCE	+0.008	+0.010	+0.028	+0.018	0
FINAL	79.009	+58.026	-58.026	-40.461	+40.439



$$\sum M_A = 0 = 58.03 + 79.0 - V_{ba}(18) = 0$$

$$V_{ba} = 7.61 \text{ k}$$

$$\sum M_B = 0 = 40.4 - V_{cb}(18) = 0$$

$$V_{cb} = 2.24 \text{ k}$$

$$\Delta = 9.85 \text{ k}$$

MEMBER END MOMENTS $(M = M_c + (\frac{P}{L})M_0$

$$M_{AB} = 1132.771 + (\frac{39.33}{9.85})(79.009)$$

$$M_{BA} = -326.503 + \text{''} (58.026)$$

$$M_{BC} = +326.503 + \text{''} (-58.026)$$

$$M_{CB} = 74.199 + \text{''} (-40.461)$$

$$M_{CD} = -74.199 + \text{''} (+40.439)$$

- 1. $M_{AB} = 2250 \text{ k-ft}$ (+)
- 2. $M_{BA} = 494 \text{ k-ft}$ (+)
- 3. $M_{BC} = -494 \text{ k-ft}$ (-)
- 4. $M_{CB} = -498 \text{ k-ft}$ (-)
- 5. $M_{CD} = 498 \text{ k-ft}$ (+)
- 6. $M_{DC} = 0 \text{ k-ft}$

← END M.

Incorrect due to w/o sidesway values!