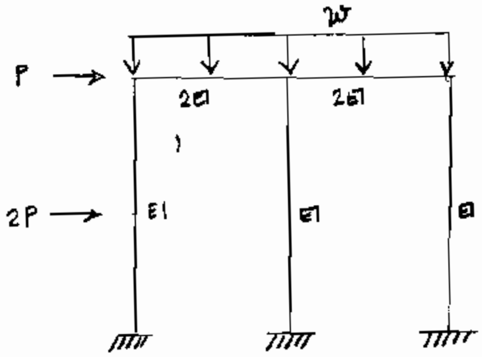
75/100 — [redacted]

CE 461 – Structural Analysis
Final Exam

Thanks for all your hard work this semester, and best of luck in your future studies and professional endeavors. It has been a pleasure having each of you in this course. If you would like your final exam grade and your final course grade emailed to you, please indicate so here: *Y / N*

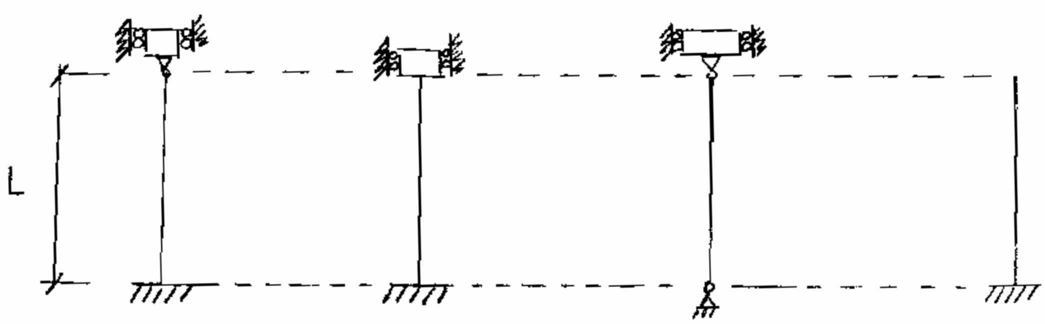
3 1. (3 pts.) Which method of indeterminate analysis that we covered in class would probably be the most *efficient* choice (least computationally intensive) to determine support reactions for the following frame? You only have a pencil, calculator, and paper as tools with which to solve the problem. Your choices are: **(1) Force Method, (2) Stiffness Method –Slope Deflection,** and **(3) Stiffness Method – Moment Distribution**. Support your answer. (You will not receive any credit without sufficient reasoning.)



Stiffness Method - Slope Deflection

- Force method is long for structures more than one or two degrees indeterminate ✓
- Moment Distribution with sidesway requires two tables & solving for R & Q forces (lots of work) ✓
- Slope Deflection would require solving simultaneous equations, but it would be the [illegible] case ✓

4 2. (4 pts.) Please supply the effective lengths for each of the following columns.



Le = 0.7L ✓	Le = 0.5L ✓	Le = L ✓	Le = 2L ✓

3 3. (3 pts.) You have been asked by your boss to design a column of prescribed length L. The material and cross-section dimensions have been chosen for you by the architect. Your preliminary design shows that a pin-pin column of that length, material, and cross-section will buckle under the given loads. What can you do as the engineer-of-record to increase the capacity of the column without changing the overall length, material, or cross-section dimensions? Please be specific.

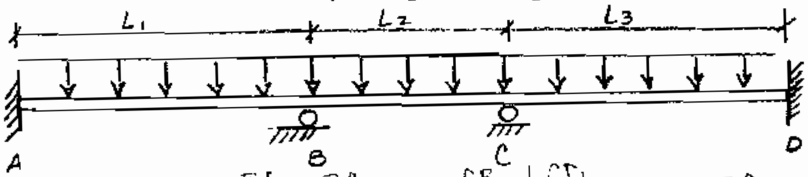
Change the supports of the column to either fix-pin or fix-fix. This will decrease the effective length of the column, raising the load needed to make the column buckle without changing the overall length, material, or cross-section dimensions. ✓

(Problem 4 got deleted ..)

0 5. (3 pts.) What does a distribution factor in the moment distribution procedure physically describe?

The distribution factor is the percent of the fixed-end moment that gets transferred through a member to the joint on the opposite end. X

8 6. (8 pts.) Fill in the following moment distribution table given the following beam, distribution factors, and fixed end moments. Carry out your precision to 0.10 ft-kip. You may not need to use all the rows in the table to successfully complete this problem.



COF = 0.5 for all members

	AB	BA	BC	CB	CD	DC
D.F.s.	—	0.444	0.556	0.556	0.444	—
FEMs	+78.1	-78.1	50	-50	78.1	-78.1
Balance Jts		12.48	15.62	-15.62	-12.48	
Carryover	6.24		-7.81	7.81		-6.24
Balance Jts		3.47	4.34	-4.34	-3.47	
Carryover	1.74		-2.17	2.17		-1.74
Balance Jts.		0.96	1.21	-1.21	-0.96	
Carryover	0.48		-0.61	0.61		-0.48
Balance Jts.		0.27	0.34	-0.34	-0.27	
Carryover	0.14		-0.17	0.17		-0.14
Balance Jts		0.08	0.09	-0.09	-0.08	
FINAL MEMBER END MOMENTS:	+86.70	-60.84	+60.84	-60.84	+60.84	-86.70

3 7. (4 pts.) For the beam in Problem 6, write the complete, numerical slope-deflection equation for M_{AB} if the support at B is subjected to a 4" upheaval due to soil swell. EI = CONSTANT

Assume L_1 is in inches, & write the equation in terms of L_1.

$$M_{nf} = \frac{2EI}{L}(2\theta_n + \theta_f - 3\psi) + FEM_{nf}$$

$\theta_A = 0$ +

$\psi = \ominus \frac{4}{L}$

$FEM_{AB} = +78.1$

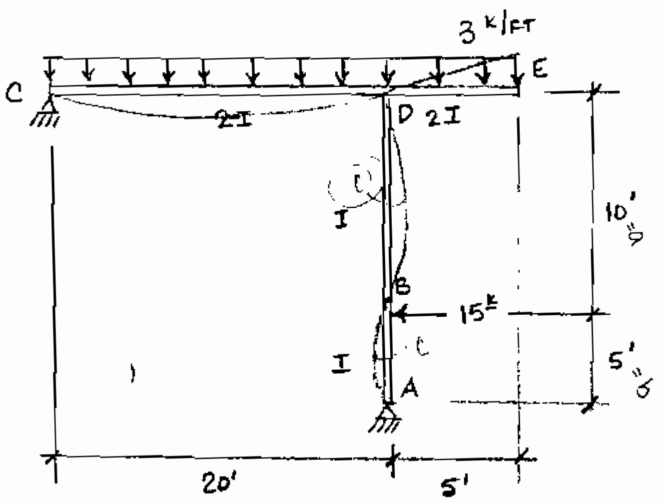
$$M_{AB} = 86.70 = \frac{2EI}{L}\left(2(0) + \theta_B - 3\ominus\frac{4}{L}\right) + 78.1$$

$$M_{AB} = 86.70^{ft\text{-}k} = \frac{2EI}{L}\left(\theta_B + \frac{12}{L}\right) + 78.1^{ft\text{-}k}$$

8. Given the following indeterminate frame,

(10) (a) (22 pts.) Solve for the member end moments using either the **slope-deflection method** or **moment distribution method.** Please begin the problem by stating which approach you have chosen.

(8) (b) (8 pts.)Draw the shear and moment diagrams for member AD Be sure to draw moment positive on the compression-side of the member.



E = CONSTANT

Use slope-deflection method

1° of freedom: θ_D

(other joints can rotate but aren't used as unknowns in eqns)

Fixed-end Moments

$FEM_{CD} = \frac{wL^2}{12} = \frac{3(20)^2}{12} = 100^{ft\cdot k}$ (+)

$FEM_{DC} = (-)\ 100^{ft\cdot k}$

$\cancel{FEM_{DE} = \frac{wL^2}{12} = \frac{3(5)^2}{12} = 6.25^{ft\cdot k}}$ (+)

$\cancel{FEM_{ED} = -6.25^{ft\cdot k}}$

-5 $FEM_{DE} = 37.5$

$FEM_{DA} = \frac{Pab^2}{L^2} = \frac{15(10)(5)^2}{15^2} = 16.667^{ft\cdot k}$ (+)

$\cancel{FEM_{AD} = -16.667^{ft\cdot k}}$ -2

Member-end Moments

$M_{CD} = 0$

$M_{DC} = \frac{3E(2I)}{20}\left(\theta_D - \cancel{\psi_{DC}}^{0}\right) + \left(-100 - \frac{100}{2}\right)$

$M_{DC} = 0.3EI\,\theta_D - 150$

$\cancel{M_{DE} = \frac{3E(2I)}{5}\left(\theta_D - \cancel{\psi_{DE}}^{0}\right) + \left(6.25 - \frac{(-6.25)}{2}\right)}$

$\cancel{M_{DE} = 1.2EI\,\theta_D + 9.375}$ -5

$M_{ED} = 0$

$M_{DA} = \frac{3E(I)}{15}\left(\theta_D - \cancel{\psi_{DA}}^{0}\right) + \left(16.667 - \frac{(-16.667)}{2}\right)$

$M_{DA} = 0.2EI\,\theta_D + 25$

$M_{AD} = 0$

Equilibrium Equation for Jt D

$M_{DC} = (0.3EI\theta_D - 150)$

$M_{DE} = (1.2EI\theta_D + 9.375)$

$M_{DA} = (0.2EI\theta_D + 25)$

$$\circlearrowleft \Sigma M_D = (0.3EI\theta_D - 150) + (0.2EI\theta_D + 25) + (1.2EI\theta_D + 9.375) = 0$$

$$1.7EI\theta_D - 115.625 = 0$$

$$1.7EI\theta_D = 115.625$$

$$\theta_D = \frac{68.015}{EI}$$

Compute Member-end Moments using θ_D

$\boxed{M_{CD} = 0}$ ✓

$M_{DC} = 0.3EI\left(\frac{68.015}{EI}\right) - 150 = -129.6$

$M_{DE} = 1.2EI\left(\frac{68.015}{EI}\right) + 9.375 = 90.993$

$\boxed{M_{ED} = 0}$ ✓

$M_{DA} = 0.2EI\left(\frac{68.015}{EI}\right) + 25 = 38.603$

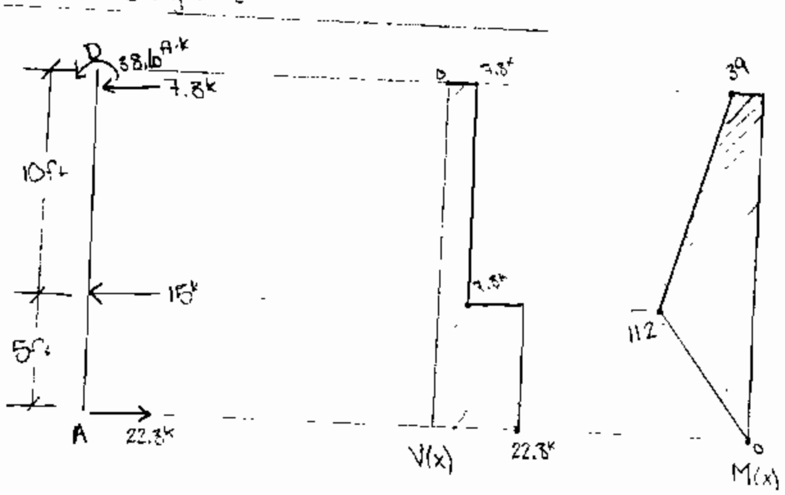
$\boxed{M_{AD} = 0}$ ✓

$\boxed{M_{DC} = -130^{ft\cdot k}}$

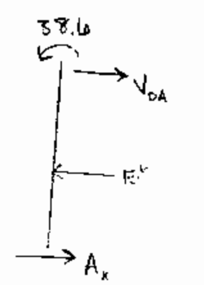
$\boxed{M_{DE} = 91.0^{ft\cdot k}}$

$\boxed{M_{DA} = 38.6^{ft\cdot k}}$

Shear & Moment Diagrams for Member AD



FBD for AD



$\circlearrowleft \Sigma M_D = 0 = 38.6 - 15(10) + A_x(15)$

$A_x = 22.3 \rightarrow$

$\rightarrow \Sigma F_x = 0 = V_{DA} - 15 + 22.3$

$V_{DA} = -7.3 \leftarrow$

$\theta_c = V_c^*$

$Y_c = M_c^*$

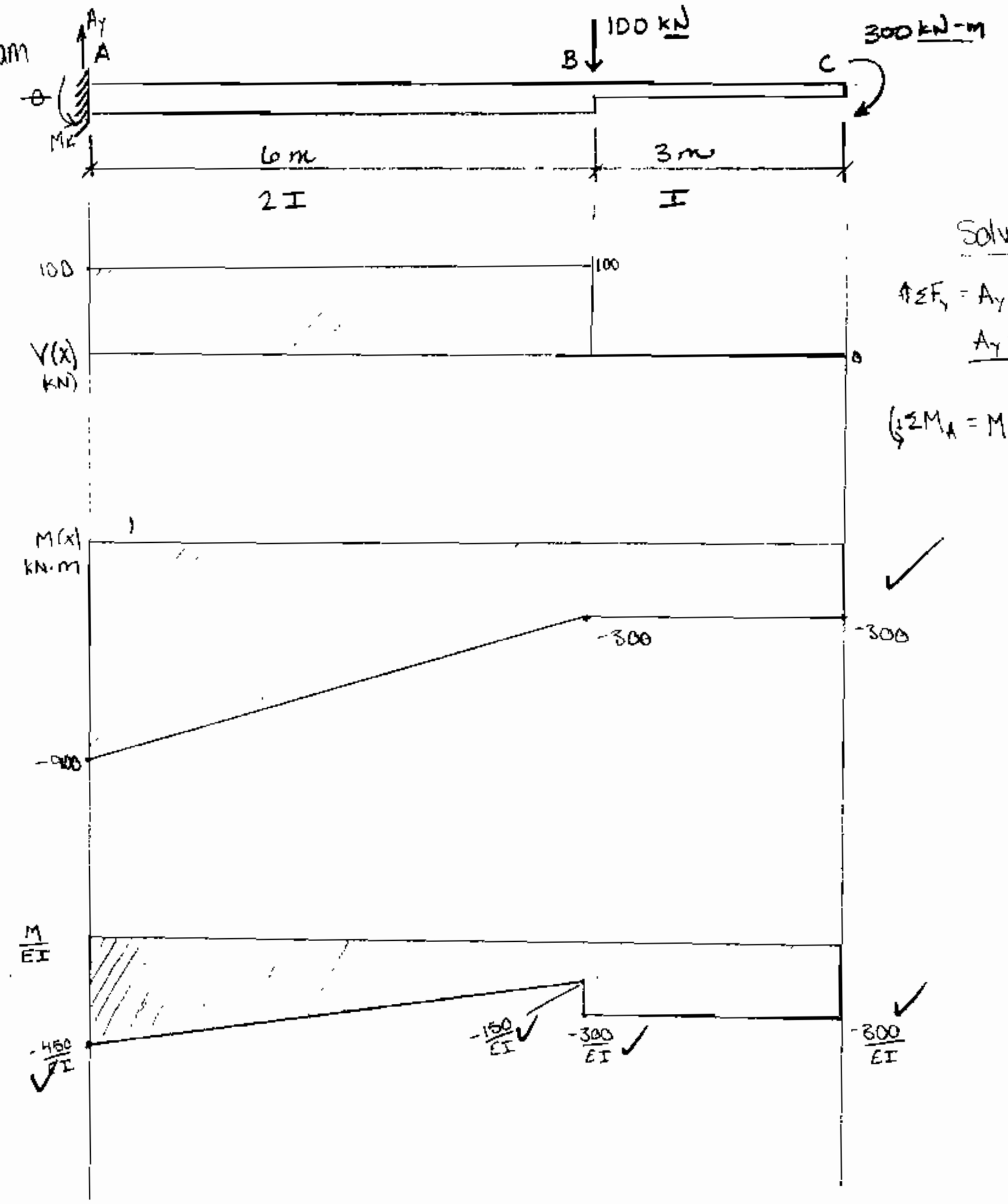
23

9. (25 pts.) Solve for the deflection at point C in the following beam. You may choose from one of the methods listed below.

GOOD

a. **Conjugate Beam Method**
b. **Castigliano's Second Theorem (Method of Least Work)**

Conjugate Beam Method



E = constant = 70 GPa

$I = 500(10^6)\ mm^4$

Solve for reactions

$\uparrow \Sigma F_y = A_y - 100^{kN} = 0$

$A_y = 100\,kN \uparrow$

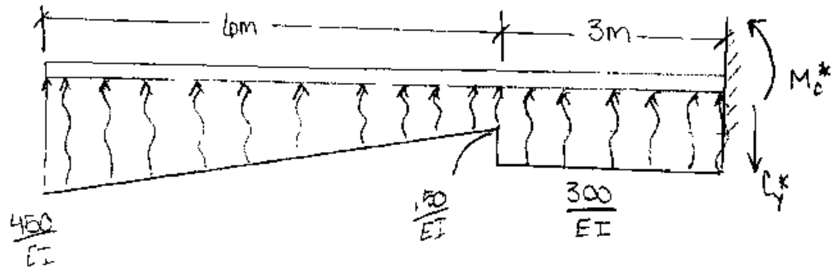
$\Sigma M_A = M_A - 100(6) - 300 = 0$

$M_A = 900\,kN \cdot m$

1 GPa = 10^9 Pa

$$EI = (70GPa)(500 \times 10^6\ mm^4)\left(\frac{10^6\ kPa}{1\ GPa}\right)\left(\frac{1m}{1000mm}\right)^4 = 3.5(10^7)\ kPa \cdot m^4$$

Conjugate Beam Loaded w/ $\frac{M}{EI}$ from real beam



Solve for reactions

$+\uparrow \Sigma F_y = \frac{150}{EI}(9) + \left(\frac{1}{2}\right)\left(\frac{300}{EI}\right)(6) + \frac{150}{EI}(3) - C_y^* = 0$

$C_y = \frac{2700}{EI}^{(kN \cdot m)(m)}$

$\circlearrowleft \Sigma M_c = M_c^* - \left(\frac{150}{EI}\right)(9)(4.5) - \left(\frac{1}{2}\right)\left(\frac{300}{EI}\right)(6)(3+4) - \left(\frac{150}{EI}\right)(3)(1.5) = 0$

$M_c^* = \frac{13050}{EI} = Y_c$

$\boxed{Y_c = \frac{13{,}050}{EI}}$

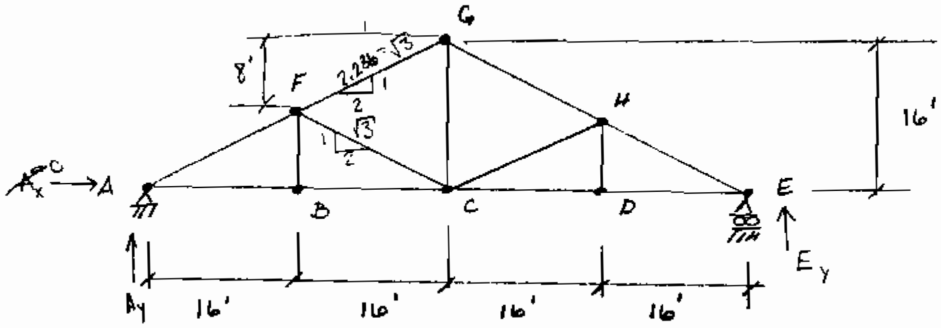
$\boxed{Y_c = 3.73(10^{-3}) \text{ rad}}$

(-1) UNITS for EI are off.

RAD? should be meters... (-1)

10

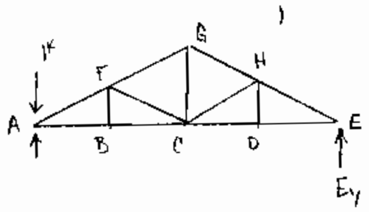
10. (20 pts.) Draw the influence line for the force in member CF for the following determinate truss.



Unit Load at A

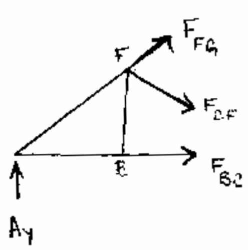
$A_y = 1^k$

$E_y = 0$



Make a cut through FC - solve for F_{CF} in terms of A_y

Unit Load to the right of C:



$\circlearrowleft \Sigma M_C = A_y(32) + \left(\frac{1}{\sqrt{3}}\right) F_{FG}(16) + \left(\frac{2}{\sqrt{3}}\right) F_{FG}(8) = 0$

$32A_y + 18.475 F_{FG} = 0$ $\quad F_{FG} = -1.732 A_y$

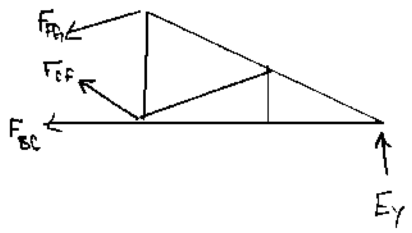
$\uparrow \Sigma F_y = A_y + \left(\frac{1}{\sqrt{3}}\right) F_{FG} - \left(\frac{1}{\sqrt{3}}\right) F_{CF} = 0$

$A_y + \left(\frac{1}{\sqrt{3}}\right)(-1.732A_y) - \left(\frac{1}{\sqrt{3}}\right) F_{CF} = 0$ $\quad F_{CF} = 0$

cancel

Unit Load to the Left of F

-solve for F_{CF} in terms of E_Y



$$\circlearrowleft + \Sigma M_C = E_Y(32) + \left(\frac{2}{\sqrt{13}}\right) F_{FG}(16) \cancel{+ F_{FG}(y)} = 0$$

$$32E_Y = 18.475 F_{FG}$$

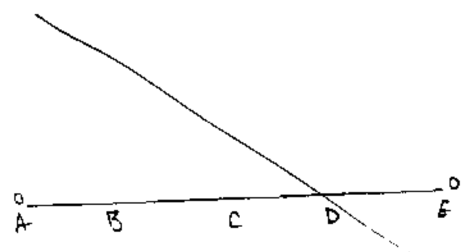
$$F_{FG} = -1.73 E_Y$$

$$\uparrow \Sigma F_Y = E_Y + \left(\frac{1}{\sqrt{13}}\right) F_{CF} - \underbrace{\left(\frac{1}{\sqrt{13}}\right)\left(-1.73 E_Y\right)}_{-E_Y} = 0$$

$$F_{CF} = 0$$

$\boxed{F_{CF} = 0}$ always X

IL for F_{CF}:



Steps

1. Make a cut through the member CF ✓
2. Solve for F_{CF} in terms of A_Y & E_Y ✓
3. Solve for A_Y & E_Y when a unit load is moving across truss ✓
4. Compute F_{CF} for each point. Connect w/ straight lines

Extra Credit No. 1 (3 pts.)
Describe the role of the following fictional figure with regard to the slope-deflection method.

(Green Giant)

The Green Giant hold the ends of each member to create Fixed-end-Moments even if the ends aren't really fixed.
FEM's are used for solutions in the Slope-Deflection method

3

Extra Credit No. 2 (2 pts.)
What's the magic number? ☺

2